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Foreword

The issue of modalities is probably one of the basic philosophical issues and has been addressed by many philosophers at many times. The contemporary notion of modality is very broad. It covers notions like necessary or possible truth and falsity, knowledge, provability and many others. The original philosophical issue deal with modalities concerning truth-values – necessity and possibility. And so they and of course their philosophical and logical treatment will be our issue as well. Because the notions of necessity and possibility concern truth and falsity we shall call them alethic modalities. Note that the notion of truth is closely related to reality. If a statement is true, there should be something in the world (no matter what a world is) that makes it true. So as a result our beliefs about what is true and what is false are also beliefs about what there is. Similarly beliefs about alethic modalities are also beliefs about what is in fact necessary and possible. The notion of alethic modality is still too broad for us for there are different types of it. One aspect of any theory of modalities is that it gives us answers to questions about what is possible and what is not. But when we say possible do we mean physically possible, metaphysically possible, logically possible or do we mean something completely different? Our topic will be the broadest notion of possibility – the logical one.

Already the first philosophers noted that not all statements are true or false in the same way. There are statements that hold no matter what happens in the world. Other statements are true but we can imagine them being false. Yet another cannot be true at all. The question which are which (and thus what is possible and what is not) and what does in fact make the statement possible or necessary is one of the basic philosophical questions. If we can answer it, we would get deep insight into reality. And because many philosophical notions are connected with the notion of modalities (e.g. essential properties) we would also have answers to many philosophical problems. Maybe therefore were alethic modalities always of great importance to philosophers. Philosophy further often transcends what is actual and makes claims not only about what there is and what it is like but also about what must be and how it must be like. Therefore many of its claims have modal force. But claims with modal force cannot be supported but by arguments employing modal notions. As a result many philosophical arguments, claims and defenses employ modal notions. There are, however, many ways how to understand that something is possible or necessary. Therefore it is important for philosophers to find a precise analysis of modalities that would shed light on these dark notions. And systems that provide such analysis will be another of our main issues.

In the last century, with the rise of formal logic and analytical philosophy, modalities have become (at least for a while) philosophical issue number one. Many logicians devised systems of modal logic that should formalize the notions of necessity and possibility and thus serve as tools of logical and philosophical analysis. For many reasons these first systems of logic were purely syntactical. It took another maybe fifty years before formal semantics was simultaneously delivered by Kripke, Hintikka and others around 1963. The most famous became Kripke's proposal operating with the notion of possible world. It has shown that the notion of possible world is a very useful one and that possible world semantics is not only suitable for precise analysis of modalities but also of many other issues in logic and philosophy (propositional attituded, counterfactuals, probabilities). This semantics was of course formal (it made use solely of set-theoretical objects like sets) so another task was to give it some intuitive interpretation so it could serve as "real" model of modal notions. From 1970's many philosophers delivered possible world semantics and its philosophical interpretation, and the discussions of it filled the main philosophical journals. Among the thinkers who joined this project we can find S. Kripke, D. Kaplan, R. M. Adams, A. Plantinga, D. M. Armstrong, D. Lewis and P. Tichý. Because there are many formal systems of modal logic, many ways how to provide them with semantics and because the issue of possible worlds is related to many complex philosophical issues (naming, essentialism, individuals etc.), the discussions resulted in many different systems. It seems to us as if the main aim of these proposals were the explanatory power and technical elegance and that the required ontology was sometimes accepted too lightheartedly as a matter of trade-off for the theoretical benefits. This lead to many brilliant systems with very controversial ontological background. But unlike in the case of formal semantics not every philosophical conception of possible worlds is equally good. The reason is that it involves application of philosophical principles that can be argued for or against independently, e.g. from non-modal cases. Such principle can be for example the treatment of quantifiers. Type of quantifier and its range in the possible world semantics should be similar or same as in the lower predicate calculus, i.e. in non-modal cases. In our study we shall therefore prefer the philosophical and ontological aspect.

This study has an ambition to cover some basic group of philosophical issues on modalities and their representation by modal logic based on possible worlds semantics. In Chapter 1, which should serve as the introduction to the problems of modalities, we focus on defining the basic modal and related notions. We try to define and distinguish different types of modalities and distinguish the notion of modality from other notions that are often confused with it. This is of utmost importance for it is often neglected in literature. Many authors present their views on what is necessary and possible without mentioning which kind of modality do they have in mind. But different types of modalities are governed by different principles. So it is difficult to evaluate some proposal without knowing which modality does it concern. Many perplexities also arise from confusion of modality and the notions of analyticity or aprioriticity. Here also we must carefully distinguish and not equate modality with one or other of these notions.

Chapter 2 concentrates on the debate on meaningfulness of quantified modal logic which is a remnant of the "wild" development of early modal logic. Modal logic experienced quick and productive development of its syntactical part. So the first we had were uninterpreted modal calculi and only after that logician tried to search for some reasonable semantics. Early proposals of such semantics were of course problematic in many ways. The main problems were quantifying into modal contexts, commitments to essentialism and possible collapse of modal logic into standard lower predicate calculus. This gave rise to the view that modal logic and the whole conception of modality as sentence operator on both closed and open formulas are irreparably flawed and as such should be either abandoned or essentially rebuilt. Most influential proponent of such view was probably W. V. O. Quine. In Chapter 2 we therefore address, discuss his objections and defend modal logic against them.

Chapter 3 represents the logical or technical view on modal logic. It shows how (some standard) systems of modal logic are formed and how we can provide models for them. There are two main upshots of this chapter. Firstly, it should provide to the reader a basic overview on how logicians see modal logic and how formal semantics can be provided. Secondly, it shows that there are many technically equivalent ways how to form a modal system and that from the standpoint of logic there is no criterion of how to choose. Conclusion to be drawn from this chapter is that if we want to search for the "best" system of modal logic we have to provide it first with some intuitive interpretation and then compare this interpretation with our philosophical views and intuitions. The reader can also observe that almost all systems of modal logic that can be found in literature are unsuitable for complex analysis of some reasonably big fragment of the natural language used for expressing our beliefs about necessity and possibility. (By reasonably big we mean language that include among others names, pronouns and definite descriptions.) All proposals take advantage of either omitting some essential part of the natural language (usually names, definite descriptions or both) or of some unintuitive assumptions (e.g. that there is name for every individual or that all non-denoting definite descriptions denote some random but particular object).

Chapter 4 focuses on the philosophical interpretation of formal semantics where the main task is not to provide the simplest modal system with greatest expressive power but to establish some basic principles according to which such system can be constructed or chosen. Main discussed issues are non-existing and non-actual objects, individuation of individuals and trans-world identity. We will argue for actualism, non-qualitative essences of individuals, absolute rather than relative notion of possibility and necessity and Kripkean view on names. Equipped with these principles we then try to evaluate some conceptions of possible worlds. We will start with combinatorial account of D. Armstrong and consider a chain of modifications which will lead us to ideas of P. Tichý and E. Zalta.

The whole study then gives a valuable overview of the (in the Czech Repblic slightly neglected) very important topic of modalities and their analysis. It does not focus on the presentation and defense of one particular system but tries to focus on philosophically important issues that are often ignored or underestimated. It also stresses the point that it is not the expressive power and technical elegance but the ontological and philosophical aspect that determines which theory is better and which worse. As a result the course we propose in the end might be theoretically not as promising as those of D. Lewis or P. Tichý but we believe that it certainly has much more acceptable ontology. We are aware that no system of modal logic can have completely intuitive ontology (especially when our intuitions differ in many respect), but we should do the best to make it as uncontroversial as possible.

1. Introduction

1.1. Possibility and Necessity Exposed

1.1.1. Types of Modalities

The issues of necessity and possibility have long philosophical tradition and they are as old as philosophy itself. It is not difficult to recognize that sentences, which we utter in our day-to-day life as well as in science or philosophy, are of two types in respect of being true. Some of the them, such as

(1) 5+7 = 12

(2) Bachelors are unmarried men

(3) If all men are mortal and Socrates is a man, then Socrates is mortal

are said to be necessarily true. The intuition behind this is that they are actually true and they would be true under any possible circumstances. Sometimes it is claimed that they are true independently of facts or only due the meanings of words or simply due to their form. Unlike those, there are sentences like

(4) Prague is the capital of Czech Republic

(5) Marcus weights 160 pounds

that are called contingent. The idea behind this is that, although they are actually true, things could have been otherwise. In other words, their negation is conceivable. Further there are dual notions to those of contingency and necessity which apply to false sentences – mere possibility and impossibility. Into the first group would fall those, that despite their actually being false could hold if the things were different. Good examples might be

(6) John might have been 5 inches taller

(7) Philadelphia could be the capital of the USA.

These three groups of sentences: necessary, contingent and merely possible are together called possible in broader sense[1]. Last group form such sentences that are necessarily false, that means false under all circumstances that could arise. We call them impossible sentences.

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[1] When we work with modal notions we have to be aware of certain ambiguity in using terms ‘possible’ and ‘contingent’. When applying ‘possible’ to sentence s it is sometimes meant that s is actually false, but could have been true. We shall call this mere or narrow possibility. On the other hand, possible are called all sentences that are not impossible. This covers all that are necessary, contingent or mere possible. In the course of our work, we shall use the term ‘possible’ in this broad way and indicate the narrow usage by ‘mere possible’

We also have to distinguish between contingent sentences, those which are actually but not necessarily true (narrow sense) and sentences that are contingently true or false including both merely possible and narrowly contingent sentences. Here we will use contingent in the narrow sense and indicate the other usage whenever needed.

When approaching modalities, some of the mentioned notions can be taken as basic (in our case necessity) and some can be derived. So impossible sentences are negations of necessary ones.[2] Sometimes merely possible sentences are also defined as negation of contingent ones.[3] And it is probably all we can say at this place. For the thing is that it is very difficult to give any precise account of what is actually meant by these terms, without entering a system of circular definitions. As we have seen, even in the intuitive explanation above, we have already employed modal verbs like could and might, which in fact indicate modal contexts and situations. As Plantinga says: "we must give examples and hope for the best."[4]

<table>
<tr><td rowspan="2">True</td><td rowspan="3">Possible - broad sense</td><td>Necessary</td></tr>
<tr><td>Contingent
(contingently true)</td></tr>
<tr><td rowspan="2">False</td><td>Merely Possible or possible in narrow sense
(contingently false)</td></tr>
<tr><td>Impossible</td><td>Impossible (necessarily false)</td></tr>
</table>

Table 1 … Overview of modal properties of sentences concerning their truth or falsity

Nevertheless, there are quite a few things we can say about modalities. First of all, we have seen that they apply to sentences. Here we have to be more precise. They do not really apply to sentences, but propositions. By proposition we can in a preliminary way understand an abstract nonlinguistic entity that is expressed by or is a content of a sentence. Propositions, as contents of sentences, are nevertheless distinct from sentences themselves. So for example the sentences 'The snow is white' and 'Schnee is weiss' express the same proposition, namely that the snow is white.

Further there are several types of modalities, which have to be distinguished. Since the modalities are usually interdefinable (impossible = negation of necessary, possible = negation of contingent), we will use in our demonstration only the notion of necessity. The most obvious type

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[2] Someone might object that there are impossible sentences like 'George Bush is a tree' that are not negations of any necessary sentence. Our reply to this is that what we have in mind here is the concept of logical necessity and impossibility. From this point of view it is not (logically) impossible, i.e. logically inconsistent, that G. Bush is a tree. (There is certainly a consistent model in which G. Bush is a tree and many other things are as we know them.) On the other hand, if we consider our sentence to be necessary then we employ rather a metaphysical kind of modality than logical. But if it is (metaphysically) impossible that G. Bush is not a tree, then it is (metaphysically) necessary that he is not. See further in this chapter for explanation on different kind of modalities.

[3] See for example Haack (1978), p. 170.

[4] Plantinga (1982), p. 1

of necessity is the logical necessity in a narrow sense (we will call it narrow logical necessity). Truths of logic, e.g. those of propositional calculus, are necessary in this sense. These truths, such as (3), are necessary only due to their form and due to meanings of logical connectives. However, this conception of necessity would be too narrow, for we consider propositions like (1) to be necessary true as well. Therefore we introduce a second type of modality – the broadly logical necessity. Into this group would fall the truths of mathematics, set theory, but also the important claims such as (2) or

(8) No one is taller than himself

(9) No philosophers are prime.

Sentences in this group are necessary mainly in virtue of meanings of their extra-logical components, e.g. terms or concepts.

Besides these two logical necessities (and their correlative notions of possibility), we can find many other propositions that are necessary or impossible in a weaker and wider sense. Commonly cited categories are metaphysical, natural (nomological) and causal necessity.[5] Let's take the natural necessity as an example. According to it, it is certainly necessary that any thrown physical body falls to the surface of Earth with an increasing speed. However, this is not necessary in the logical sense, for there could have been no gravitation on Earth or the whole mechanics of space could function in a different way. There is of course massive controversy about physical necessity and whether it exists at all. We will nevertheless leave this issue aside, for our main concern is the logical necessity.

Regarding the different types of modalities, one more thing has to be mentioned, namely that all of them are somehow "dependent" on the concept of narrow logical necessity and logical entailment. As M. Tooley writes "[…] a state of affairs is nomologically impossible if its non-existence is logically entailed by the laws of nature. […] And a statement is analytically necessary if it is logically entailed by meaning postulates, or by relations of synonymy."[6] And similarly in other cases. When we want to explain what does some kind of modality mean or how do we understand it we usually do so in terms of compatibility with (or being entailed by) some set of propositions. We do not want to say that it is the only option but it is certainly the preferred and broadly used one. But that means that we can barely make any theoretical analysis of other types of modality, unless we resolve at least some of the puzzling problems concerning the necessity and impossibility in the strictest sense – the logical one. This dependence of course does not affect the

[5] There is of course no uniformity and agreement about which types of modality there are and whether they are really independent. This concerns for example metaphysical modality. It is in no way obvious, whether this kind of modality, usually considered as broader than logical but narrower than natural modality, can squeeze between them or whether it coincides with either one of them. Also there are some debates concerning the relation of natural and causal modality. But none of those issue is of our concern here.

intuitive level of our thinking. We can, of course, better understand the notion of natural necessity, because we have better knowledge of natural laws than laws of modal logic. And we can certainly say lot of interesting things about it. But as soon as we want to deliver some precise analysis of what is naturally possible and what is not, we have no other way but to examine whether certain proposition about certain natural fact is compatible with (our conception of laws of) physics or not. That means in the end that if we cannot coherently say what does it means that something is logically necessary or possible, then it is probably of no use to interpret the other broader types. And maybe it would be of no use to talk about them seriously at all.

1.1.2. What We Cannot Mix

The more difficult it is to explain what we mean by 'necessary' and 'possible', the more important it is to distinguish these concepts from others that might coincide with them in some cases. The idea of necessity that we have elaborated so far is the one that attaches to propositions and modifies their being true or false. These types of modality, including logical, nomological, causal modality and maybe some other kinds, are sometimes called alethic modalities.

Necessity cannot be however confused with the notion that Plantinga calls " 'unrevisability' or perhaps 'ungiveupability'"[7] According to him "Some philosophers hold that *no* proposition – not even the austerest law of logic – is in principle immune from revision. The future development of science (though presumably not that of theology) could lead us rationally to abandon any belief we now hold, including the law of non-contradiction and modus ponens itself."[8] It is of course Quine and his "Two Dogmas of Empiricism" that Plantinga has in mind. Quine imagines the world of beliefs as a spider's web. Some propositions are closer to the center of the web (truths of logic or set theory) some are closer to the periphery (e.g. our empirical beliefs). And no matter how close to the center a truth is it can still be abandoned or revised, if it is rational or practical to do so. But if it is possible for a truth to be revised, it is certainly not necessary.

We must, however, still distinguish between a proposition's being necessary and someone's readiness or unwillingness to give it up. A proposition is necessary in virtue of its ontological status, to wit, possession of certain property. But it is always an individual or a group of people who revise or give up certain truths. In that sense every truth is revisable for there always can be someone who thinks that it is no truth. That, of course, does not imply that a given truth is not a necessary one. Paraphrasing one of Plantinga's examples, I might be ready to give up some basic mathematical

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[6] Tooley (1999), p. vii
[7] Plantinga (1982), p. 3

truths in order to simplify the Christian doctrine of Trinity. But my willingness to sacrifice mathematics has nothing to do with the status of its truths for it is not popularity or number of adherents that constitute the necessity of its truths. So for example Quine is unwilling to revise the lower predicate calculus or the principle of extensionality. Moreover, he defends it as we will see later against any extension towards modal logic. (Thus he betrays in a way his own pragmatic approach, for intensional logic has proven to be very useful tool when doing analysis of natural language.) Some other scholars favor intensional logic and revise the principle of extensionality. But this attitude of different people does not affect the principle's being true or false on its own. Moreover, "a proposition might be necessarily true even if most people thought it false or held no opinion whatever on the matter."[9] Similar argument works in case of the attempt to paraphrase 'necessary' in a way of 'what cannot be rationally rejected'.

Another issue, distinct from the necessary – contingent problematic, is the one of a priori, a posteriori and self-evident truths. Firstly, we have to stress the fact that this is an epistemological issue not a metaphysical or logical one. Therefore a priori and a posteriori attaches in the first place to knowledge. The standard account is that person S knows proposition p a priori if S's knowledge is absolutely independent of experience. (A posteriori knowledge is then defined as one that depends on some kind of experience.) In this sense p is a priori relative to particular person or to his (act of) knowledge. In a secondary sense we can say that a proposition or truth is a priori in general. Then we would say that a priori truth is such that it can be known independently of any experience. This, however, causes many problems and questions. Firstly we have to ask who is the subject in question, to whom can this truth be known. We would have to ask whether there is someone who actually knows it a priori, or is it enough that there could be someone who knows it a priori or must all people know it a priori? The other move would be to say that we are talking about idealized thinker. But would we mean God or some idealized finite mind? Moreover in our explication, we have used the word "can", which indicate that there is another kind of modality in question. However, it is not clear after all, what kind of modality it might be. Because of these puzzling questions, to which there are no common answers, it seems to be better to stick to the particular pieces of knowledge of particular people.

The problem is even more puzzling, because we cannot switch the "can" used in the definition of a priori for "must". We cannot say that a priori truth is such that it cannot be possibly known empirically. It is because (almost) every a priori truth can be known a posteriori. So for example truth of every theorem of mathematics can be known to person S on the basis of consultation of a more educated colleague. Kripke's example is the believing that a number is prime

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[8] Ibid.
[9] Ibid., p. 4

on basis of machine computation.[10] If a computer tells me that certain number is prime, my belief is based on an a posteriori justification. Nevertheless, there could be someone who made some effort and did the computation and as a result knows the same truth in an a priori manner.[11]

Again, we want to show, that although necessary and a priori can coincide in some cases, in general they are two distinct things. First of all, it is no doubt, that there are necessary truths that are not known a priori by anyone. An often used example is Fermat's last theorem (or its negation, if it is false). It is necessarily true or false, but nobody knows therefore nobody knows a priori. One can object that it can be known in principle. But can we be sure? Kripke comments on this nicely: "Maybe there is a mathematical proof deciding this question; maybe every mathematical question is decidable by an intuitive proof or disproof. Hilbert thought so; others have thought not; still others have thought the question unintelligible unless the notion of intuitive proof is replaced by that of formal proof in a single system. Certainly no formal system decides all mathematical questions, as we know from Goedel."[12] So maybe there is really no proof for our theorem, not even in principle. On the other hand it is sure that at the time being nobody knows whether it is true thus nobody knows it a priori. Moreover, as we have already indicated, any truth of mathematics (a necessary one in broader logical sense) can be known either a priori (by performing the demonstration or by computing) or a posteriori (consulting some handbook, table or more educated friend).

Now what about the converse? Is it true that from the fact that something is known a priori it follows that it is necessary? There is some supporting intuition. If it is known a priori then it would seem to be known independently of the way things are. So it should be necessary after all. Here we will help us with Kripke again who, discussing the same issue, says: "If it [the knowledge] depended on some contingent feature of the actual world, how could you know it without looking? Maybe the actual world is one of the possible worlds in which it would have been false. This depends on the thesis that there can't be a way of knowing about the actual world without looking that wouldn't be a way of knowing the same thing about every possible world. This involves problems of epistemology and the nature of knowledge; and of course it is very vaguely stated."[13] Even if it does not answer the question, it shows that the question is not a trivial one. It shows that

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[10] Kripke (1996), p. 35

[11] The issue of a computer proof and a priori knowledge is however not a simple one. Because it is not our main issue, we will not address it at the length that it deserves. The only thing to be mentioned here is that it is of course clear that there are certainly ways of knowing the computer-based results in an a priori mode. So for example the programmer who wrote the code for the prime number recognition program or a computer expert certainly have an a priori justification for the result. But in the case of an ordinary user (which covers most of the situations) without any particular knowledge of the algorithms used the justification seems to be rather a posteriori, similar to the one we get from newspapers or television news. For those who do not favor this view a sophisticated defense of the claim that knowledge based on computer-based computations is or can be a priori can be found e.g. in Tyler Burge's paper "Computer Proof, Apriori Knowledge, And Other Minds" (Philosophical Perspectives 12 (1998), pp. 1 - 37).

[12] Ibid., p. 37

[13] Ibid., p. 38

we cannot ague that something is necessary just on the basis that it is known in a certain way, namely a priori. There might exist a posteriori necessary truths[14]. There are, of course, disputes among philosophers, which combinations of epistemic and alethic modalities there are. Nevertheless our example with Fermat's last theorem should be sufficient to show that the notion of necessity and a priori do not coincide and therefore are distinct.

The last distinction to be made is the one between necessity and a semantic notion of analyticity; for some philosophers like Carnap have believed, that analyticity could be a foundation for concept of logical necessity. (After Quine's attack in Two Dogmas this conception has been abandoned.) We have already mentioned analytic necessity above, but the problem requires closer look. In general the concept is blurred, but we can surely give some examples of analytic truths. When trying to explain the concept of analyticity, we can say that an analytic proposition is such that it is (in some sense) true only in virtue of its meaning or meanings of its parts, and it is true in that manner in all possible worlds. Synthetic propositions (the dual "opposing" concept) can be said to be true in virtue of "facts". A contemporary paradigm of analytic statements is "All bachelors are unmarried men". We can determine its truth-value in any possible world by simple inspection of meanings of involved words. In our case, the meaning of the predicate "unmarried man" is already "included" in the meaning of "bachelor". On the other hand, a synthetic statement such as "Some dogs are white" requires us to investigate into facts and look for white dogs in the (respective possible) world. The meanings themselves do not help.

Similar to previous distinction between a priori and necessity we want to show that necessity and analyticity are different concepts. This time it will not be so easy, because the concept of analyticity and its relation to necessity is very unclear. (If the concept of analyticity is not empty as Quine argues). However, quoting Quine, the relation of analyticity and necessity can be expressed by definition of analytic truth: "Statements which are analytic by general philosophical acclaim [...] fall into two classes. Those of the first class are logical truths; those of the second class can be turned into logical truths by putting synonyms for synonyms."[15] So apparently concept of analyticity is broader than the one of logical necessity. And for our present purposes it might be sufficient to say that if something is analytic, it is both necessary and a priori. Necessary in virtue of quoted definition, a priori, because if a statement is true in virtue of some relations among meanings of its parts (here synonymy), then these relations are not parts of the physical world. A theoretical justification is always needed.

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[14] In the modern time, idea that there are necessary a posteriori truths is generally ascribed to Kripke who discusses it on different places of Naming And Necessity such as pp. 35-8, 158-60. Nevertheless, I think, he does not give any particular example of it.

[15] Quine (1953), p. 22

Prior to moving on let us sum up the results of this exposition. We have shown that there is more than one type of modality that we employ when using the language. Therefore we have to carefully distinguish in every case, which is the modality in question and handle it accordingly. In the following chapters, it is going to be the broad and narrow logical necessity that we will focus on, while we will disregard the problems of natural or causal modalities or others. We have also sketched a difference between several conceptual pairs: necessary/possible, analytic/synthetic, a priori/a posteriori. Our aim was to show that even though they might sometimes overlap they are in general distinct and have to be carefully distinguished.

1.1.3. De Re And De Dicto

We have introduced modalities as properties of propositions that can affect their way of being true or false. This way of ascribing modality is sometimes called modality de dicto, for it is the proposition – dictum – to which the modality is attached. Let's take for example a true statement

(10) Whatever is seen sitting is sitting

And make a necessary statement out of it by applying the necessitation to it. If what we get is the statement

(11) It is necessarily true that whatever is seen sitting is sitting

In this case we have used ascription de dicto and we have got a true statement. On the other hand, if we got statement

(12) Whatever is seen sitting is necessarily sitting

we have used the modality de re (ascription of necessary property directly to the thing in question) and, which is important, we have got a false statement. For even though I see now that Peter sitting, he could have been actually standing or doing something completely else. So we can apply the necessitation in two different ways and get two statements with opposite truth-values.

It is Aristotle that is said to have as a first one noticed this distinction, which has played an important role in philosophical analysis ever since.[16] And it has been probably the most misleading distinction in the analysis of statements containing modal terms.

There is usually little dispute among philosophers about modality de dicto. We understand pretty well, what such modalities mean and we even have sound logical systems that are able to provide tools for analysis of statements containing them. The other modality, de re, is nevertheless problematic. What we want to assert by it (as in (12)) is that a thing in question could not lack the property, come what may. That means we have to start distinguishing among the properties those, that a thing could lack (accidental properties) and those that it cannot (essential properties). This distinction leads then to a very complicated debate and no commonly accepted solution has been presented yet. Moreover no reliable tool for analysis of de re modal statements has been presented, even though many have tried.[17]

1.2. Why modal logic?

We have seen so far, that the topic of modalities is not a simple one. We saw that we have to distinguish several types of modality. And there are many other puzzles such as those of rigid designators, identity and many other that are especially tricky in modal contexts. But they are philosophically very interesting and fruitful. However, a philosopher who wants to study necessity and possibility, unlike a physicist studying the outer world, has a great disadvantage. He cannot go simply out and observe the reality, derive from it some basic rules, build his theory and than test it in practice. The empirical scientists can found or verify their theories on observing something existing. For modalities, this does not work. Everything, that is present here and now is simply there. No modality is involved in it. But which among the present things and state of affairs are necessary, if any? Or could something be possibly otherwise? Could, for example, there be a talking donkey? There are no answers to be found in the world alone.

We have already mentioned that modalities can be soundly treated as traits of propositions. The intuition behind is that it is the way we imagine, speak and above reason about things and states of affairs that employs modal concepts.[18] Moreover, there is the apparent distinction between necessary and contingent truths in the area of theoretical sciences. Therefore if we want to learn something about modality, there is probably no other means of study than to observe the language, the way we speak and reason and the way, in which modalized sentences are built and used. However, for such a mission, we need a powerful tool. A tool that would allow us to unveil the role that modalities play in our talk and in the same time provide us good means of precise analysis.

The standard tool for philosophical analysis in the last century has become logic. After enormous development in the beginning of 20th century, it was developed enough to be suitable for analysis of various kind of philosophical statements and arguments as well as for analysis of

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[16] Plantinga in (1982) quoting an article of W. Kneale shows a modal figure from Prior Analytics, I,9, which is valid only if read as de re. The distinction of de dicto and de re has however become a commonly used tool in theological and philosophical discussion. See for example Aquinas' Summa Contra Gentiles, I, 67, 8 or Summa Theologica I, 14, 13.
[17] See Wright (1951) Chapter 3.B for one of the proposals.
[18] This does not deny that there might be some seeds of modality in the things themselves, such as essential properties. But let's leave the question open for now.

fragments of natural language. Moreover, it was the linguistic turn in philosophy that definitely buried the platonic vision of language[19] as a mere vehicle of knowledge and let it play important role in philosophical inquiries about world, experience, ethics and many other issues. So the modern philosophical logic was founded and analysis became preferred philosophical style.

But why is logic so important to philosophy? Nice answer is delivered by P. Tichý in "Sinn & Bedeutung Revisited". He says: "It would be utmost interesting if someone could clarify the role, which arguments play in philosophical debates. In science, the role of arguments is rather limited. Science founds its results on observation and experiments – not that much on logic. In philosophy, on the other hand, logic is the only tool. A philosopher has either a valid argument or he has nothing; his proposals depend on arguments by which they are supported."[20] If we would like to draw a moral out of it, it could read: "No philosophy without logic."

Although it is not commonly accepted that philosophy would be reducible to logic adherents of more analytical style of writing point out that logical analysis can help philosophers in sweeping away many confusions brought about by surface form of language and arguments and throw more light on how our knowledge works. The side effect is clearer philosophical style. And modal logic, an extension to classical propositional or functional calculus that allows handling statements containing modalities, is exactly the case.

For how else shall we facilitate our talking about necessity and possibility when no distinction can be drawn at the level of what is actual? The most promising way is to collect our intuitions and try to produce some viable formal theory. Then we can translate our claims and theories into it and see what type of modality is really involved, what consequences our claims and theories have and into which extent do they satisfy our initial intuitions. Because sometimes the theory or thesis itself looks harmless, but what it entails can be unacceptable.

An the end of this section let us show a situation where logical analysis and employment of modal logic or its semantics may be (and in fact is) useful. It is example from medieval philosophical debates about God's foreknowledge about future contingent things. In Summa Theologiae St Thomas Aquinas tries to defend a view that God's foreknowledge is compatible with our intuition that event in the future are in principle undetermined. A corresponding view would be that statements about future have at the time being no truth value. In the beginning Aquinas formulates a set of objections one of them as follows: "Further, everything known by God must necessarily be, because even what we ourselves know, must necessarily be; and, of course, the knowledge of God is much more certain than ours. But no future contingent things must necessarily

[19] For elaboration see Fløistad (1981), p.3.
[20] Tichý (1996a), p. 147, my translation.

be. Therefore no contingent future thing is known by God."[21] The key to Aqinas' answer is a logical distinction between de dicto and de re modality applied to the statement

(13) Everything known by God must necessarily be.

He says: "Hence also this proposition, 'Everything known by God must necessarily be,' is usually distinguished; for this may refer to the thing, or to the saying. If it refers to the thing, it is divided and false; for the sense is, 'Everything which God knows is necessary.' If understood of the saying, it is composite and true; for the sense is, 'This proposition, 'that which is known by God is' is necessary.'" Now what exactly Aquinas did. He realized that the modality used in the objectors statement (13) is ambiguous. So (13) could in fact mean either (let p stand for a proposition)

(14) It is necessary that: for every p, if God knows p, then p (is true)

or

(15) For every p, if God knows p, then p is necessary (is necessarily true).

Using the possible world semantics and some standard system of modal logic we can then clearly evaluate (15) as false. For the meaning of (15) is something like

(15') given some proposition p and particular possible world w_0, if God knows p in w_0, then p is true in all possible worlds.

On the first sight it is clear that (15') is too ambitious. It reaches far beyond standard logic of knowledge.[22] For how can a factual knowledge guarantee that the thing known holds in all possible worlds (even in those, where God maybe does not exist – this is of course not one of Aquinas thoughts). (14) on the other hand seems acceptable, for it says

(14') for every possible world w and for every p, if God knows p in w, then p is true in w.

This reading is intuitively acceptable, moreover it blocks the objection against Aquinas. Because the reading the objector's would need is: (i) given p_0, if God knows p_0, then p_0 is necessarily true and (ii) God knows p_0, therefore p is necessarily true. But all he gets in premise (i) is: it is necessary that given p_0, if God knows p_0, then p_0 is true. And that is certainly not enough to establish the modalized conclusion. So we (and Aquinas) can finally reject the objection as ill-founded.

[21] Aquinas (1964), Ia, 14, 13, objection 3

[22] Standard inference in logic of knowledge is that from the fact that A knows p one can derive p, but certainly not necessarily p.

2. Modal logic attacked and defended

2.1. Do We Need Modal Logic?

I have argued above that modal logic is an important tool for philosophers. But is it good for any other purpose? Some philosophers, e.g. Quine, who worship classical extensional logic and dislike its extensions and revisions, claim that it is not. We could for example ask, whether modal (or any other non-classical) logic and its features, namely the analysis of modal reasoning, can be of any help in science. However, Quine would tell us that if we can pursue science without any non-classical logical systems, there is no need to implement them. And for Quine it is obvious that we can do so. Moreover, according to him, formalization of informal arguments and logical analysis of language in general have in the end no other purpose than production of high-quality scientific language. So, if the modal logic is not needed in science, it has no application at all. But is it really the case?

First of all we have to ask whether scientific usage of logical tools for 'regimentation' of informal thinking is really the only one. It is no doubt true that a production of precise scientific language (language devoid of for example non-referring singular terms or homonymous expressions) was one of important motivations of founders of modern logic, such as Frege. (Actually, the need for such a project was advocated long before Frege by Leibniz.) But it certainly was not the only motivation. Philosophers and logicians quickly realized that there are many other task where logical analysis might be useful, e.g. detection of philosophical pseudoproblems.

As we have shown, logic can play an important role in philosophy in general. Some logicians, moreover, find it useful to construe a precise language that would (besides scientific talk) accommodate moral, theological, fictional discourse or everyday reasoning. It would certainly be a priceless achievement to find a formal model of our moral systems and to show which are its axioms and how every moral requirement can be proved or disproved in it. Further a question arises, whether the language of science itself is really devoid of modal contexts. On the first sight it does not seem to be so. It is indisputable fact that in science we use such terms as dispositions (fragile, soluble etc.) To formulate the view let's take the expression 'soluble'. Solubility is, so the view, adequately explicated by a conditional. In our case to say that X is soluble is to say that if X were placed in water, x would dissolve. However, the only way of analyzing it in the system of classical logic is to use the material conditional. But such analysis is inadequate. For standard material conditional ($\varphi \rightarrow \psi$), a truth-function defined as

φ	ψ	φ→ψ
T	T	T
T	F	F
F	T	T
F	F	T

Table 2…Truth table for material conditional

does not satisfy our intuition. It is due to the fact that when φ is false the conditional is trivially true. So the interpretation of solubility in the way of material conditional would be ‘if X is placed in water x dissolves’ would also have this feature. The conditional will be true even in the case that X will never be placed in water. But this is not right. We certainly do not think, that we can truly predicate solubility (in water) on the account that the object in question has never been in water before. Therefore we would need some other analysis, presumably the one with subjunctive conditional.[23] But subjunctive conditionals are analyzed by modal notions.[24] So in the end of the day we have to say that even in science, we cannot make do without modal notions.

Quine of course wants to show that he can, even in the problematic cases of dispositions and subjunctive conditionals, make do without modal notions. With S. Haack we can sum this up by saying that Quine denies thesis that “certain locutions are (i) essential to scientific discourse and (ii) inexplicable except of modal terms.”[25] In paragraph 46 of Word and Object his strategy is as follows. First he excludes from the scientific language subjunctive conditional and the dispositional operator ‘-ble’ “as a freely applicable ingredient of canonical notation.”[26] This is defendable because such ingredients can be expendable in scientific talk. So let’s accept it. What Quine allows us is to keep one by one any general term (soluble, fragile etc.) no matter how dispositional or subjunctive it is. The only prerequisite is that this term is in a sense well-behaved, that means that there is some ‘stabilizing factor’, which is in our case some kind of subvisible structure. Further Quine argues that there is in fact no substantial difference between dispositional and non-dispositional general terms. General terms like ‘soluble’ or ‘fragile’ show their dispositional character only in linguistic or etymological sense. “It is only on this etymological count, if at all,

[23] Statements like ‘If I were you I would do it otherwise’ or ‘If Turks reached Prague Czechs would speak Turkish today’.
[24] To be more precise it is the today accepted analysis of subjunctive conditionals that uses modal notions. Although we accept it as correct, it does not exclude the (no matter how unlikely) possibility that someone will come up with a better analysis without modal notions. For the modal analyses see Stalnaker (1968) or D. Lewis (1973).
[25] Haack (1978), p. 179
[26] Quine (1960), §46, p. 225

that such a term as 'red' might not be said to be dispositional as well. An object is red if it is disposed, given a chance, to reflect a certain range of low frequencies selectively. Redness of things is like solubility in that the pattern of subvisible structure concerned now happens to be fairly well understood…"[27] So it is the theory of subvisible structure that provides good 'stabilizing factor' for dispositions. According to this theory what we express by 'soluble' is that a given object has structure suitable for dissolving. And by 'red' we say exactly the same, namely, that the object is so structured that it reflects certain frequencies. So after all, there is nothing like genuine dispositions, or in other words properties and dispositions are in fact on a par. Our criterion for distinguishing them is only the etymological form (suffix '-ble' etc.) and the fact that the respective subvisible structure is not that well known and understood as the one of regular general terms. Using his theory of subvisible structure Quine proposes to explicate etymological dispositions relatively to etymologically non-dispositional terms (soluble – dissolve) without using subjunctive conditionals. Then he introduces a slightly problematic relation 'alike in molecular structure' marked M. Using this relation he explicates 'x is soluble' as 'there is an object with similar molecular structure and it dissolves' with tenseless verbs. Schematically '(∃y)(Mxy & y dissolves)'.

So Quine claims that he delivers an explication of dispositions without using modal notions. He is of course aware that it is vague and approximate but it fulfils "likely purposes of the original sentence"[28] and it gives a hint for general procedure. Quine's analysis is however not devoid of problems. Even if we admit (undefined) relation M and give it some rational meaning, there will remain plenty of dispositions, in case of which the required internal structure is not known at all. An example given by Haack is irritability. Moreover this analysis will probably not do the trick in case of (i) things that are one of a kind, e.g. universe and (ii) things with dispositions that have never been manifested before. In case (i) the problem is clear. Quine's analysis includes besides the given thing x one more object y with certain properties. In order to avoid triviality object y should be different than x. So the analysis requires that there are at least two distinct objects with similar molecular structure such that of one we already know that it possesses properties in question. So if there is only one object of given kind, the analysis fails. So statements like "The Universe is expandable" which should be true (because universe, at least given current theories, really expands) would be false. Concerning (ii) we can take as an example nuclear power plants. "[…] all nuclear power stations have a disposition to explode in certain circumstances, even though, thus far, safety precautions have ensured that those circumstances have not arisen, so that none have ever

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[27] Quine (1960), §46, p. 223 f.
[28] Ibid., p. 224

exploded"[29] It is certainly nonsense to claim that a nuclear power plant lacks that disposition just because there is no such power plant that already exploded.

There is one more reason why not to be satisfied with Quine's analysis. Quine claims that all dispositions are dispositions only in etymological sense and that, once the relevant molecular structure is known well enough, they can be reduced to some non-dispositional terms. By saying this he seems to hint that given any dispositional term t it is only question of time, when the needed subtle structure will be discovered and t will be paraphrased to some non-dispositional term. After the discovery, like in the case of 'red', no reference to irreducible disposition will be needed. Dispositions are, so to speak, only a façon de parler. In this respect Quine's views seem to be question-begging. First of all, the idea that once in future all internal structures accounting for all dispositions will be found presupposes something like a "completed" enterprise of science. And "one may well feel discomfort with the appeal to a finished, by contrast with an ongoing, science, for such a distinction fits especially ill with Quine's usually pragmatic approach to philosophy of science."[30] Second of all it might be well the case that the structure required to perform Quinian explication of disposition by non-dispositional terms will be so complex that it will be never discovered. Remember our example of irritability. Here we can at least hope in the infinite progress of science. But what if the structure of reality itself is such that we will be left with genuine irreducible dispositions forever. This will be the case on any non-deterministic view of reality. And the nowadays accepted quantum mechanics seems to be one of those.

So after all we have to say that Quainian approach is by no means unproblematic and that it does not deliver any reliable way of how to omit subjunctive conditionals from the language of science. Dispositions indisputably belong to the language of science and even if some of them might be in some cases explicable by some non-dispositional terms and reference to some structural properties we are in general left with irreducible dispositions which cannot be explained but by subjunctive conditionals which means with help of modal notions. SO we have to conclude, that besides obvious usefulness of modal notions (and modal logic as an analytical tool) in the language of philosophy, there is a place for them (and modal logic) in the exact science itself.

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[29] Haack (1978), p. 180. This example is, of course, not true after the Czernobyl catastrophe in 1986. But one can surely restrict the statement to nuclear power plants of certain type or origin, e.g. nuclear power plants built by Westinghouse.
[30] Haack (1978), p. 180

2.2. *Quine's Attack*

Quine's dissatisfaction, or better to say distaste, for modal notions and modal logic (ML) might have one more important source. Systems of modal logic presented in the first half on 20th century (1918 C. I. Lewis, 1947 R. Carnap and others) were mostly developed as syntactical systems. A sound semantic side of these systems was underdeveloped and it took a long time before they were provided with some reasonable model-theoretic semantics.[31] So Quine, probably reasonably, expressed doubts, whether ML as conceived by early modal logicians, can be of any good. More precisely, it causes in his opinion more problems than it solves and as a result it is not helpful at all. And as we will see, he mostly criticizes the fact that the interpretation, not the syntactical systems alone, is fraught with difficulties. So which are the cursed features of modal logic?

His first criticism, or shall we say remark, applies to initial development of modal logic. When according to Quine Russell and Whitehead were developing their system of logic, they decided to call the truth function '→'[32] defined by table on page 17 material implication. So they would read the formula '$\varphi \rightarrow \psi$' as 'φ implies ψ' instead of more correct 'if φ then ψ'. Later on, logicians felt that this sense of 'imply' is in fact too weak to satisfy our intuition about logical implication (meant as entailment) and for this they proposed a new function, called 'strict implication' to formalize it. And first systems of ML took strict implication as a primitive concept. Now, Quine argues that giving name 'implication' to the material conditional 'if-then' caused enormous confusion. For 'if-then' is a sentential operator while 'implies' is "a verb, which connects names of statements and thus expresses relation of the named statements."[33] Because of this confusion, called by Quine confusion between use and mention[34], strict implication was developed as sentence-forming operator on a par with e.g. conjunction and not (as Quine would imagine) as semantical predicate on a par with 'is true'. As a result '□', which embodies the notion of logical validity or entailment and should have been semantical predicate similarly to 'is valid', was

[31] This view is for example shared by Haack. See Haack (1978), p. 178 for details. According to my point of view first such semantics was delivered in Kripke (1963).

[32] As a matter of convention, we will use standard logical notation lower predicate calculus. Among modal motions, we use '⥽' for strict implication (instead of more often used 'fishhook' sign), '□' for necessity and '◊' for possibility.

[33] Quine (1994), p. 165

[34] We can understand a sentential operator as a function that given n statements returns another statement. So '→' produces from φ and ψ a statement $\varphi \rightarrow \psi$. The function *uses* φ and ψ to form a new statement. On the other hand the predicate 'implies' takes as arguments names of sentences. It does not form a new statement, but predicate something about the old. Because the arguments are names, φ and ψ do not themselves appear in the 'φ' implies 'ψ' more than 'cat' appears in 'catapult'. The statements are only *mentioned*.

conceived as sentential operator. The alleged reason was the need for definition of strict implication in means of '□' and '→' as $\varphi \prec \psi =_{df} \Box(\varphi \rightarrow \psi)$.

Contrary to modal logicians, Quine is persuaded that '□' should be in fact interpreted as a semantical predicate. Moreover, any other interpretation faces serious problems. His argumentation in favor of '□' as semantical predicate goes as follows. He starts by affirming that one of the valuable and important principles of standard logic is its principle of extensionality. In order to explain it Quine introduces several notions. The first one is 'purely referential occurrence' of terms. A term in a statement has purely referential occurrence if and only if the term serves in that particular context solely to refer to its object. One criterion for referential occurrence is substitutivity of identity, that means that, if we have a sentence containing referential occurrence of term t and we have t' that refers to the same object, then t can be substituted by t' without changing the truth-value of the sentence . So for example occurrence of 'Bob Dylan' in

(1) Bob Dylan is a famous singer

is purely referential. The reason is that it can be substituted by 'Robert Zimmerman'[35] without changing truth-value of the statement. It is plain that if something is true of Bob Dylan, it will be true of him even if we use another of his names. Contexts, in which terms have referential occurrences, are called referential (or direct). There are of course contexts that do not satisfy the criterion of substitutivity of identity such as quotation marks. So in a statement

(2) 'Bob Dylan' has 8 letters

we cannot substitute 'Robert Zimmerman' for 'Bob Dylan' without changing the truth-value of the whole. Quine calls these contexts referentially opaque (indirect). In this context the expression does no longer refers to the object in question. In our example the expression 'Bob Dylan' is used as a part of an expression referring to the term 'Bob Dylan' itself. Again an occurrence of term in such a context is also called opaque. Further an occurrence of statement in a complex statement is called truth-functional if a substitution of a given statement by a statement with the same truth-value preserves truth value of the whole compound. To illustrate this, let us take two true sentences with different meaning one of them being

(3) The Earth is round

the other

(4) Napoleon was a French Emperor.

Now, if we take conjunction of (3) and some random true statement, we will get a true compound statement. If we now in such a statement replace (3) with (4), the truth-value will not change. Therefore the occurrence of both (3) and (4) in our statement is truth-functional. Now, on the other

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[35] Original name of that famous folk-rock singer.

hand, imagine my friend James, who is a physicist, but is very weak in history (so he thinks that Napoleon is a Greek poet). Given a (for our purpose true) sentence

(5) James believes that the Earth is round

we can try to do the same trick again and replace (3) in (5) with (4). This time, however, we will get a false statement, because James does not believe that (5) is true. As a result, occurrence of (3) in (5) cannot be truth-functional. The operator 'believe that' generates opaque context.

Now Quine requires that all logical operators produce referential contexts. It is natural in the case of logical connectives such as negation or conjunction as we will see. In other words, he adheres to the principle of extensionality. Nevertheless, he admits that there is one important opaque operator, namely quotation marks. Quotation marks clearly produce opaque contexts. Moreover, they are well established in our language and they can be easily understood.[36] From this position he claims that to preserve extensionality of logic, '□' is to be read as semantical predicate attachable to names of sentences using quotation marks. On such reading, '□' itself is not the cause of opacity. It is quotation marks that are responsible for the opacity. Following his proposal we would write

(6) It is necessary that 9 is greater than 5

as

(6') □ '9>5'

and explain the departure from extensionality in it by appeal to the opacity of quotation marks. The reading would be something like 'statement '9>5' is necessarily true' where the concept of necessity would cover more or less the concept of logical implication or the one of theoremhood. This reading has several advantages. First of all we have got an easily understandable concept of modality because for interpretation of '□' we have used well known concept of validity or theoremhood. Secondly, we have avoided problems with quantifying-in. The reason is that we apply '□' to names of statements and not statements themselves. That means that we cannot get a situation where an objectual quantifier attached to modalized statement would bound variable in the scope of modal operator because all occurrences of variables in its scope are opaque. And thirdly there are no problems with interpretation of iterated modalities. Symbol '□' of object language applied to the name of statement is no more a formula in original object language, but, similarly to 'is valid', an expression of meta-language. That means, however, that '□' (symbol of object language) is no more applicable to the new modalized statement that belongs to meta-language. Therefore iteration of modalities in one language is not possible at all. Every application of '□' takes us to a new meta-

[36] The function of quotation marks is to produce from any expression the name of the expression. So using quotation marks on 'Bob Dylan', which refers to the particular individual, we get the expression ' 'Bob Dylan' ' which refers to the expression 'Bob Dylan' but no more to the individual.

language. So the issue of iterated modalities and its understanding is not solved in any way but avoided at all.

To support his way of dealing with modalities (and to show why we cannot apply '□' to anything else, e.g. statements) Quine introduces two more arguments. The first one shows that there is a technical way of getting rid of quotation marks for good using names of letters and the concatenation operator.[37] We will not reproduce the argument, but the result is that whenever we use an expression with quotation marks, we can replace it by another equivalent expression without them and thus preserve extensionality principle and get rid of opacity for good.[38] In the second one, Quine shows that we cannot depart from extensionality so easily as we might think. He argues that every referential context in which two logically equivalent expression can be substituted salva veritate (which is the case of necessity) is also truth-functional. So we cannot introduce a necessity operator that would not be truth-functional and which would produce a referential context (as required by quantification theory). Every such operator must be opaque and therefore all occurrences of terms in its scope must be opaque as well. As a result '9' in opaque context '□ 9>5' would no longer refer to the number itself, which is counterintuitive. It is exactly the number 9 that we intend to refer to when asserting the statement in question. Quine concludes that if we want to allow other operator of necessity than the semantical one, we cannot do it while preserving all principles of standard logic. In order to introduce a different version of '□' we would have to revise bigger part of logic, such as the theory of singular terms.[39]

Quine is of course aware that there are alternatives to his own view. Besides his "innocuous" treatment of necessity there are two other ways how to conceive it. So all together we can have '□' as:

(i) semantical predicate (Quine's favorite), which applies to names of sentences (as discussed above). It plays a role of a verb. '□'9>5'' is interpreted as ''9>5' is necessarily true'. It is similar to 'is valid' in proof theory.

(ii) sentence-forming operator on closed sentences (statements), which acts as an adverb. So '□(9>5)' can be read as '9 is necessarily greater than 5'.

(iii) Sentence-forming operator on both open and closed sentences. It functions like (ii) but it is possible to form sentences like '(∀x)□(x>5)'.

Now necessity in terms of (ii) or (iii) is according to Quine direct departure from extensionality. While in (i) we explained opacity of '□' by citing well known opacity of quotation marks, in (ii) we have to say that it is '□' itself that is opaque. If it operates directly on sentences, there is simply no

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[37] Cf. Quine (1994), p. 161
[38] For details see Quine (1994), p. 161-2.
[39] Ibid., p. 163-4

more place for quotation marks because application of them to sentences never produce a sentence, always a sentence-name. When reading necessity according to (ii) or (iii) and preserving other features of lower predicate calculus we get a puzzling situation where

(7) □(9>5) and

(8) the number of planets = 9

are true while

(9) □(the number of planets > 5)

is false. Apparently '□' in (7) and (9) does not produce truth-functional contexts, because the substitution of identity does not hold. But we have seen above that every referential context in which logical equivalents are exchangeable must be truth-functional. So we have to conclude (by modus tollens) that it cannot produce referential context at all and that '□' is really opaque. On the other hand, we want as we will see later that occurrences of '9' and 'the number of planets' in (7) to (9) are referential.

If we understand necessity as mentioned in (ii) Quine offers us a way out. He proposes how to transform it from statement operator to semantical one.[40] Such a transformation has, so Quine says, many advantages: "[…] it is at the semantical or proof-theoretical level, where we talk about expressions and their truth values under various substitutions, that we make clear and useful sense of logical validity; and it is logical validity that comes nearest to being clear explication of 'Nec' taken as semantical predicate. […] A fourth reason is that the adoption of 'nec' as a statement operator tempts one to go a step further and use it as a sentence operator subject to quantification. […] A fifth reason has to do with iteration. Since 'nec' attaches to a statement and produces a statement, 'nec' can then be applied again. On the other hand 'Nec' attaches to a name and yields a statement, to which, therefore, it cannot be applied again."[41] So Quine is after all quite tolerant toward using '□' as statement operator. He says that we can always convert it back to what he thinks is appropriate reading. The only problem is that once we start with (ii) we are tempted to move towards (iii). An indirect result of this view is that Quine has very few objections to propositional modal logic. As long as we do not allow '□'; to operate on open sentences, it does not really matter whether we read it as mentioned in (i) or (ii). There is always a way how to get back to (i) and from there to extensionality with quotation marks.

However, if we go for necessity as sentence operator as mentioned in (iii), then the conception of modalities is, according to Quine, fraught with serious difficulties. Quine claims that such modal logic cannot be coherently interpreted, because there are conflicts between conception of modalities, classical theory of quantification and classical theory of singular terms. Before

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[40] Ibid., p. 166-7

[41] Ibid., p.168

moving on let's expand these theories a bit. The conflicting theory of modality takes '$\Box$' as sentence-forming operator that can be applied even to open sentences. So both '$\Box\forall xFx$' and '$\forall x\Box Fx$' are well formed formulas. Of course, '$\Box$' is not a truth-functional but an opaque operator. On the other hand, the theory of quantification requires that a variable that is bound by quantificator is in the referential position. That means it must refer to the object and as a result substitution must work. So while '$\forall xFx$' satisfies this criterion, '$\forall x\Box Fx$' does not, because '$\Box$' produces referentially opaque contexts. The problem with the theory of singular terms has been already shown on the example with statements (7) – (9). It is the substitutivity requirement that is not met in modal contexts, but is required for quantification.

In his criticism of modal logic "mark (iii)" Quine raises three charges: (i) no satisfying account on quantification into modal contexts has been given so far; (ii) undesired statements are logically true, e.g. that every identity is necessary; (iii) such modal logic implies "Aristotelian essentialism".

Let's take these charges one by one. Concerning (i) let's take into account statement like

(10) $(\exists x)\Box(x>7)$.

Because '$\Box$' can now be applied to open sentences (10) is a well formed formula. Now from (10) we can get both

(11) $\Box(9>7)$,

which is true and

(12) $\Box$(the number of planets > 7),

which is obviously false. At the same time it seems to be true that '9=the number of planets', so the result in (11) and (12) is really puzzling. Either there is something wrong with quantification or the classical conception of singular terms. Quine, of course, says that there is something wrong with '$\Box$'. But we have to remember that modal contexts are, according to Quine, opaque. They are on a par with contexts produced by quotation marks. So variables in them do not play their normal referential role. So the occurrence of 'x' in (10) is similar to occurrence of 'cat' in 'cattle'. As a result when applying '$\exists$' to '$\Box(x>7)$', similarly to its application to '$(9>7)$', there are no free variables that the quantifier could bind. On the other hand, Quine requires in accordance with normal intuition about quantification that the occurrences of variables in the scope of quantificator are indeed referential. We must be able to derive from '$(\exists x)(x>7)$' both '$(9>7)$' and '(number of planets > 7)'. Otherwise the quantification would not make any sense. In a way "Quine concludes, [that] in the sentence following quantifier positions of variables have to be referential. Further names, which refer to the same object, must be interchangeable in these positions, or, in other words, whatever is asserted in this open sentence to be true of an object in our universe of discourse, must be true of it regardless of how it is referred to."[42] That basically means that indiscernibility of identicals (IID) - principle symbolized as $(\forall x)(\forall y)((x=y)\rightarrow(Fx\equiv Fy)))$ - must hold so that quantification can function in its normal way. In our example, however, it is not the case. So the whole quantification theory, as we understand it in lower predicate calculus, seems to cease to make any sense. And because Quine thinks that classical logic is correct, he thinks that the problem is the wrong interpretation of modal operators.

Further, in relation to (ii), there are two unpleasant consequences that our kind of quantified logic has. One that Quine elaborates in "Three Grades of Modal Involvement" is that necessary identity thesis (NI – $(\forall x)(\forall y)((x=y)\rightarrow\Box(x=y)))$ holds in quantified modal systems. Secondly, Quine even shows in Word and Object that under certain restrictions on universe of discourse that would allow meaningful quantification into modal contexts, '$p\rightarrow\Box p$' would be true and (because '$\Box p\rightarrow p$' is axiom of standard modal systems, we would get '$\Box p\equiv p$' and) the whole modal distinction would collapse.

Quine's final and probably most discussed remark about quantified modal logic is that it commits one to what he calls the "Aristotelian essentialism", which is the claim that "an object, of itself and by whatever name or none, must be seen as having some of its traits necessarily and others contingently, despite the fact that the latter traits follow just as analytically from some ways of specifying the object as the former traits do from other ways of specifying it."[43] Later he adds that this essentialism is "[…] the doctrine that some of the attributes of a thing (quite independently of the language in which the thing is referred to, if at all) may be essential to it, and others accidental."[44] In later papers he turns his attention into a slightly different kind of essentialism (but he does not make it explicit) In "Intensions Revisited" he discusses the notion of proper names and he adds: "A rigid designator differs from others that it picks out its object by its essential traits. It designates the object in all possible worlds in which it exists. Talk of possible worlds is a graphic way of waging the essentialist philosophy, but it is not only that; it is not an explanation. Essence is needed to identify an object from one possible world to another."[45]

In Three Grades Quine lists features of essentialism. According to him it is a doctrine that allows us to form statements like

(13) $(\exists x)(\Box Fx \;\&\; Gx \;\&\; \neg\Box Gx)$,

where 'Fx' and 'Gx' are some open sentences. Now, (13) says that there is an x such that property F is essential to it but property G is only contingent. In case of number 9 F could be 'to be bigger than

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[42] Føllesdal (1969), p. 176
[43] Quine (1953), p. 148
[44] Quine (1994), p. 176
[45] Quine (1981), p. 118

5’ and G could be ;to number the planets of solar system. In any case 9 is necessarily bigger than 5 independently on how it is being referred to (e.g. ‘9’, ‘the number of planets’ or ‘$^{2}\sqrt{81}$’). But if it is what quantified modal logic allows, then there must be something in the things themselves in which the necessity “resides”. Quine concludes that essentialism is a “metaphysical jungle” and that we can avoid it only if we interpret modalities in his semantical manner.

There is one more place where Quine nicely illustrates problems that essentialism implies. His argument is following: “Mathematicians may conceivably said to be necessarily rational and not necessarily two-legged; and cyclists necessarily two-legged and not necessarily rational. But what of an individual who counts among his eccentricities both mathematics and cycling? Is this concrete individual necessarily rational and contingently two-legged or vice versa? Just insofar as we are talking referentially of the object, with no special bias toward a background grouping of mathematicians as against cyclists or vice versa, there is no semblance of sense in rating some of his attributes as necessary and others as contingent.”[46] According to Quine we cannot deliver any sound reason that explains this puzzle nor can we draw any reasonable distinction between essential and accidental properties . It is at least the lack of satisfactory explanation of this “metaphysical” kind of necessity that makes the essentialism a dubious doctrine. And any theory committed to it is suspicious as well.

2.3. Response to Quine

In the previous chapter, we have the important parts of Quine’s argumentation against modal logic. It is quite powerful and if it were sound and unavoidable it would certainly lead to the end of the enterprise of modal logic. Nevertheless, modal logic has flourished despite Quine’s critical remarks and some people, though not too many, have joined the Quinian discussion and delivered some defense. Most proponents in fact ignored Quines’s criticism and continued to develop more and more sophisticated versions of modal logic.

Before we try to answer Quine, we have to mention one feature of the whole debate around modal notions that complicates its evaluation. It is the fact that most of the responses are based on denying Quine’s premises. There is in fact very little (or maybe none) effort to attack Quine’s course of argumentation itself. A very illustrative comment on this debate is provided by Haack. She says: “The debate runs true to form: Quine’s criticisms are answered by rejection of the premises on which they rest. Quine thinks quantifiers talk about things; according to the substitutional interpretation, quantifiers talk about talk about things. Quine things that modality is talk about talk about things; according to essentialism, modality is talk about things. Quine's views about quantification and necessity aren't sacrosanct, of course … But this doesn't make the tendency for the debate between Quine and the defenders of modal logic to degenerate into assertion and counter-assertion less disagreeable…" This section should lead us through part of the debate and some of the outcomes.

To remind us what Quine's criticism of modal notions is about, let's sum up the points once more: Firstly, Quine shows that whole modal logic was conceived in a sin - a confusion between material conditional 'if-then' and logical 'implication', which brought about wrong understanding of necessity operator as sentential instead of more adequate understanding as semantical. Secondly, he is worried about how to combine opaque operator of necessity with (according to him) standard views on quantification, substitution and singular terms. Thirdly he is persuaded that modalities are really connected only with the way we talk about things. An object has some properties necessarily and some not in relation how is it being referred to. Any attempt to say that necessity resides in things themselves (essentialism) is harmful and leads to "bewildering" results.

Concerning the first point, there is not much to say. First, it is not an decisive argument but rather an observation. The way we decide to call relations or predicates makes in the end no difference. The relation of identity for example would be still the same relation even if we decide for any reason to give it the name 'schmidentity'. When Quine complains about the confusion the only thing we can say is that modal logic simply formalizes certain relation between sentences which is stronger than what we may call material implication. What Quine wants to show is that if the founders of modal logic had recognized the difference between 'if-then' and 'implies' we would have been saved from problems typical for modal logic such as quantification into modal contexts or interpretation of iterated modalities and maybe interpretation of the whole modal talk at all. (On the other hand the pursuit of these issues turned out to be philosophically interesting and fruitful.) Quine admonishes us to correct this confusion and to use necessary as predicate of names of sentences, a semantical correlate of 'is valid' from proof theory. . Given this understanding of '□' we would have a system of modal logic in which many of problems we encounter in our systems today (iterated modalities etc.) would never arise.

There is, nevertheless, one doubt left. It is after all not certain whether this "Quinian" modal logic would be sufficient tool for the analysis of our modal reasoning. And it is really so. Quine's proposed modality is too weak to formalize all what we expect. In fact the system based on "semantical" modality would be weaker than anything that we use and want to use today.[47] So the

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[46] Quine (1960), p. 199

[47] More on this can be found in Haack (1978). Haack quotes Montague's article 'Syntactical treatments of modality' (Acta Philosophica Fennica 16), where he shows that syntactical treatment of modality would lead to systems weaker

answer to Quine would be that although his proposal has some sound core, we have to (and also want to) go for more ambitious project. For, "if we insist on equating formal logic with strongly extensional functional calculi [Quine's aim], then P. F. Strawson is correct in saying that 'the analytical equipment (of the formal logician) is inadequate for the dissection of most ordinary types of empirical statements.'"[48]

Quine's second point is far more serious because, if quantification and substitution are at stake, there must be something wrong in the very heart of modal logic. All Quine's points focus on one main problem: How can we interpret quantification into opaque contexts? Basically there are two ways: (i) we can construct our semantics in such a way that we accept Quine's thesis but avoid the consequences or (ii) we can block Quine by denying one of his premises. The first way is the way of Church and Carnap. In Church's system variables in modal contexts take intensions[49] as values and therefore they are in referential positions. So after all, there are no opaque constructions in the system and Quine's requirement is satisfied. Carnap on the other hand, admits that modal operators generate opaque contexts but argues that we can make sense of quantification and substitution if we again let variables range over intensions and impose more restrictions on unqualified substitution. As a result two expressions can be substituted for each other in any context of Carnap's system only if they refer to the same intension (i.e. are necessary co-extensional). However, there is one more explanation that Church and Carnap have to give. They would have to explain what in fact are the entities they quantify over and, as Quine would certainly require, give appropriate identity criteria. And this might be a pretty difficult task.

The second approach would be to declare that quantification functions differently than we think. An example can be some "strong" version of the so-called substitutional interpretation of quantifiers. Without going into much detail, the basic idea of substitutional quantifiers is that bound variables do not range over the objects but over expressions of language, namely singular terms. In that case there is no difference between Quinean referential and opaque contexts. The reference is always an expression, and all positions of variables are in fact referential. So there would be no problem after all. However I am not sure whether someone has ever defended this view.

As we have seen, none of the mentioned approaches really denies Quine's theses about quantification and reference. In the end, it seems that Føllesdal is right when he says that "what one has to do is to accept the thesis and still find a way of avoiding Quine's calamitous conclusion, that is, one must find a semantics of modal contexts according to which these contexts are at the same

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than S1. But the notion of necessity that we use in our modal reasoning is essentially stronger than this, so Quine's proposal seems to be too cautious and insufficient for what modal logicians want to achieve.

[48] Marcus (1993), p. 5. Quotation of Strawson is from: Strawson, F. P. (1952), Introduction to Logical Theory, Methuen, London, p. 216.

[49] The idea is that variables range over attributes instead of classes or individual concepts instead of individuals.

time referentially transparent (as required for quantification) and extensionally opaque; that is, co-extensional expressions must not be interchangeable (since such interchangeability would amount precisely to the collapse of modal distinction warned against by Quine)."[50,51] To develop such an approach one must begin by distinguishing between expressions that refer and those that have extension. On the level of singular terms it means to distinguish between genuine singular terms and those that are parasitic in a way on general terms. Usually given example is the one of names ('G. W. Bush) and definite descriptions ('the current president of USA'). The current prevailing view about (proper) names is that they refer directly to their objects. It means that no matter what change an object undergoes, the name refers to it. It also refers to it in any possible world in which the object exists.[52] A definite description on the other hand is more like a general term that is true of one object. It picks whatever object satisfies the description. So its extension can change in the same way as the extensions of general terms ('blue', 'bigger than G.W. Bush' etc.) change in time and possible worlds. Only when we split extension and reference can we achieve a situation when there will be referentially transparent contexts that will be extensionally opaque.

A nice argument along these lines has been proposed quite early after Quine's initial criticism in Smullyan (1948). Smullyan argues that if we take a system in which definite descriptions and class abstracts are defined contextually (i.e. are not primitive expressions of language) then the problems with quantification and substitution do not arise. As a premise he accepts the Russellian analysis of definite descriptions in terms of existential statements and develops an example similar to Quine's. Then he builds a variant of Quine's argument about number 9:

(14) It is logically necessary that 9 is less than 10

(15) 9 = the number of planets

(16) Therefore it is logically necessary that the number of planets is less then 10

Smullyan notes that such argument uses the principle of indiscernibility of identicals ($(\forall x)(x=y \rightarrow Fx \equiv Fy)$) and the way of reasoning is: if 9 has a property F and 9 is identical to the number of planets then the number of planets also has the property F. Let's accept it for a while and take a

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[50] Føllesdal (1969), p. 179 f.

[51] Føllesdal uses following definitions:
A position of a singular term (general term or sentence) in an expression is referential (extensional) if and only if any singular term (general term or sentence) that occurs in that position can be replaced by a co-referential (co-extensional) expression without the containing expression changing its reference (extension).
A construction, or mode of containment is *referentially (extensionally) transparent* if and only if for every expression which may be an ingredient in the construction, every position which is *referential(extensional) in the ingredient* expression is *referential(extensional) in the product* of construction; otherwise it is *referentially (extensionally) opaque*.

[52] The basic idea of theory of proper names (sometimes called rigid designators) is that the semantical relation between the expression and the object is constant and direct and is not mediated by any third entity. Once the reference of the proper name has been set by what Kripke calls "initial baptism", the name always (in all contexts) refers to the same thing. Nice reasons for introduction of proper names (rigid designators) are in Føllesdal (1998), p. 106.

different example. Provided that James thinks of the number 3 we can then consider two statements about James:

(17) There is one and only one thing, which James is now thinking of and which is necessarily odd,

(18) It is necessary that there is one and only one thing that James is now thinking of and it is odd.

On the first sight we can say that (18) is false. The reason for saying that is that James could have been thinking of anything else, e.g. the number 34 or his lunch. (17) on the other hand is true. There is really a thing which is necessarily odd and as a matter of brute fact (contingently) James thinks of it. Now, let's get back to our initial argument. We can see that (16) has form similar to (18) and as such does not follow from (14) and (15). So the argument (14) – (16) is fallacious. What we could get out of (14) and (15) is the statement

(19) As a matter of brute fact, the number of planets satisfies the condition, that it is necessary that x is less than 10.

The problem is, however, not in the principle of indiscernibility of identicals. If we try to put down the initial argument in logical notation (using $(\iota x)(\varphi x)$ as abbreviation for definite description), we will get following

(20) $\Box(Fy)$

(21) $y = (\iota x)(\varphi x)$

(22) $\Box(F((\iota x)(\varphi x)))$

Without using much logic we can see that this argument is not valid. (21) is not a true identity statement; it is rather (using Russell's analysis) existential statement of form

(21') $(\exists x)(\varphi x \;\&\; (\forall z)(\varphi z \rightarrow x=z) \;\&\; x=y)$.

So premise (21) is a contingent existential statement (saying that there is a unique thing that has property φ and is identical to y) and the necessary conclusion does not follow. What we in fact get as a conclusion is something like

(23) $(\exists x)(\varphi x \;\&\; (\forall z)(\varphi z \rightarrow x=z) \;\&\; \Box Fx)$.

which corresponds with (19) in our example with James. In order to get (22) we would need instead of (21) a stronger premise

(21'') $\Box(y=(\iota x)(\varphi x))$

saying that the identity between the object y and the object sorted out by the description is necessary or in other words that both terms are necessarily co-extensional[53]. Only given (21'') can

[53] This indicates another possible approach to the problem. We could claim that, no matter how we define descriptions, (21) is not an identity but some weaker equivalency claim. In our case, the equivalence can be co-extensiveness. It is

the argument be conclusive. But that shows that Quine's original argument that should compromise modal logic does not hold.

Quine's second problem with substitution and quantifying was that from a true existential statement '(∃x)(□(x>7))' we can derive using concretisation both true '□(9>7)' and false '□(the number of planets > 7)'. To resolve this we need to draw a distinction between genuine names and definite descriptions. '9' is then treated as a proper name (rigid designator) and therefore can be used as a substitute of x in modal (referentially transparent) context. 'The number of planets' on the other hand is a definite description and is (contingently) co-extensive with 9. (By the way, it could have been co-extensive with 7 if some celestial catastrophe had happened.) But '□' generates extensionally opaque context, so the substitution of 'the number of planets' for x may change truth value of the statement.

So after all it is not the principle of indiscernibility of identicals itself that causes the problems but rather the idea that names and definite descriptions are the same type of expression and with more importance the idea that the class of singular referring expressions coincides with the class of expressions with extension. Quine would probably defend his view and say that names and descriptions are not that different. After all, every proper name can be assimilates to contextually eliminable definite description (his famous move in "On What There Is"). So his thoughts about singular terms in fact wipe off the distinction that Smullyan used to reject second premise of Quine's argument and that we have used above to resolve the concretisation problem. Here one might reach a conclusion that the debate about this particular issue or even about modalities in general is basically a conflict of intuitions about certain metalogical issues, e.g. about function and types of singular terms. But even if it were so, it seems that Quine's premises or intuitions are really problematic (if not false) and as such have to be rejected.

It is easy to see that definite descriptions (defined contextually or else) and names cannot be assimilated. To illustrate it let's take a different example. It might seem that in

(24) G. W. Bush is the current President of USA

we express identity of two individuals named by 'G. W. Bush' and 'the current President of USA'. And it is also the view of Quine. But it is not clearly so easy. While we are ready to say that 'the current President of USA' could have denoted Al Gore instead of G. W. Bush if things were different, we are certainly not ready to say it about the term 'G. W. Bush'. Moreover the current

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typical feature of extensional logical systems that they equate some weaker equivalence with identity. So for example interconnection of identity and indiscernibility in principle called Leibniz's law may be one such weakening of genuine identity. It is O.K. as long as we are concerned with spatiotemporal objects in extensional system, but may start to be insufficient in modal contexts where new dimension has to be added. So in modal contexts genuine identity must be used in order to guarantee substitutivity. And it is, of course, no longer connected with indisceribility. So the course of agrument (14) – (16) is blocked. This view is precisely developed in "Modalities and Intensional Languages" in Marcus (1993), pp. 5 – 23.

President of USA was formerly R. Reagan, but G. W. Bush certainly wasn't R. Reagan at any time of his existence. (The former excludes the possibility of arguing that we can make descriptions rigid by adding the word 'actual' or so. There is always the time aspect that betrays us there.) Therefore 'G. W. Bush' is not simply identical with 'the current President of USA'. In our talk we used so far, the expressions are contingently co-extensional but certainly not co-referential. So (24) is in fact not an usual identity statement. And therefore, no matter how we formalize this idea, we have to distinguish in our logical system between proper names ('G. W. Bush') which both refer and have extension and definite descriptions that describe (or sort out) some unique object but do not directly refer to it. And Quine fails to this distinction that then leads him astray when he tries to think about modal contexts. Before we move on I would like to address a potential criticism. It can be argued that even though 'G. W. Bush' is not equivalent with our definite description, there is a specific definite description that it is equivalent to. The only possibility would be to invoke individual essences which is not open to antiessentialist like Quine or to use Quine's own trick from On What There Is and come up with the definite description 'the only x that G-W-Bushizes'. (Let's leave aside the tricky question how to determine the extension of our new predicate.) But if it is really so and all names are equivalent to definite descriptions then it also concerns the name '9' in his argument (14) - (16). The second premise (15) would then be a statement of form

(25) $(\iota x)(\varphi x) = (\iota y)(\psi y)$

which is

(26) $(\exists x)(\exists y)(\varphi x \;\&\; (\forall z)(\varphi z \rightarrow x=z) \;\&\; (\psi y \;\&\; (\forall z)(\psi y \rightarrow y=z) \;\&\; x=y)$.

But (26) is also not sufficient to derive the required conclusion. The only situation when the original argument holds is when both expressions in (15) are treated as genuine proper names. And that correctness of such a view can be easily challenged.

Last point that troubles Quine is the one of essentialism. He claims that in order to make any sense of modal logic, we must accept what he calls "Aristotelian" essentialism. First, we have to clarify what he really means. If we consider the quotations on page 26, we have to conclude that there are at least two versions of essentialism that he addresses. In his earlier papers (first and second quotation), the essentialism he criticizes says no more than an object has some properties necessarily and others contingently, independently on how it is being referred to. If we compare both quotations we see that whereas the first says that there are essential properties (Quine uses indicative), in the other Quine merely states that there may be some. So the claim is that our modal logic requires necessity to reside in things themselves (or there must be something in them that gives foundation to our talk). As we can see from the second quotation, the actual existence of necessary properties is not required but probably presupposed. (Why would we otherwise deploy modal logic, right?) Let's call this notion of essentialism weak.

However, in the last quotation on page 26 originating from a later paper, Quine turns against the notion of essence as employed in the discussion about trans-world identity. In the discussion about identification of object across possible worlds, some people claimed that objects must possess unique essential properties (essences), by which they can be identified. But the implied notion of essentialism is now much stronger than the weak before. This strong version claims that not only are there essential properties, but for every object there also is such essential property that the object has it and nothing else does.

No matter how obscure the latter doctrine is, the problem is that Quine does not make it explicit that he is criticizing a different notion. He never distinguishes the weak and strong version in his papers (he also never criticizes them both at the same time) and so confuses them. Probably therefore most of his opponents defend modal logic against commitment to the strong (or some stronger) version of essentialism than the one we called weak. Also the title "Aristotelian" is misleading because Aristotle certainly demanded more than the weak notion.[54] But when Quine was committing modal logic to essentialism he certainly meant the weak one.[55] After all, as Føllesdal argues, "we should keep in mind that both of the philosophers who loomed large in Quine's criticism of modal logic, Carnap and C. I. Lewis, championed quantified modal logic while at the same time they rejected as metaphysical nonsense the traditional Aristotelian view that necessity inheres in things and not in language."[56] In his essays Quine only reminds those logicians that quantification into modal contexts brings this "Aristotelian" view back into the game.

Let's conclude then that it is the weak notion that Quine has essentially in mind. But this weak notion is harmless to modal logic. It is not more objectionable than the modal operator itself. Moreover any operator that creates referentially transparent but extensionally opaque contexts (counterfactuals, probability, believe-that contexts) requires this kind of essentialism, because our weak notion is nothing more than the combination of extensionally opacity (modal distinction of properties) and referential transparency (independently on the way of referring). It is even questionable if such a view should be called essentialism at all, because if it says no more that necessity resides in things (or there is something that corresponds to it in things themselves), then we are all pretty much essentialists.

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[54] So far I know Aristotle not only required that the modality resides in things themselves but also defended the view that membership in certain natural kind is an essential property of a thing.

[55] Føllesdal in (1998) explains Quine's confusion in means of "misconceived idea that essential properties are required in order to 'keep track of the objects over which one quantifies as one moves from one possible world to the next'" (p. 105). Føllesdal nevertheless thinks that essences are not needed at all, because variables and proper names are rigid designators that simply refer to the same thing no matter what. He adds: "In my view, the notion of a genuine singular term is not fundamentally modal notion; it is not a notion that requires appeal to necessity or essentialism […] That genuine singular terms refer to the same object 'in all possible worlds' […] is not definitive of such terms. It merely follows from the fact that these terms are referring expressions. Preservation of reference is basic to our use of language even outside of modal logic[…]" (p. 105) So Quine has been simply mistaken.

[56] Føllesdal (1998), p. 104

Independently of what Quine says there are of course stronger versions of essentialism that might be philosophically dubious. The versions may be characterized by following schemata:

(27) $(\exists x)(\Box Fx \;\&\; \neg\Box Gx)$

(28) $(\exists x)(\exists y)(x \neq y \;\&\; \Box Fx \;\&\; \neg\Box Fy)$.

(29) $(\exists x)(\exists y)(x \neq y \;\&\; \Box Fx \;\&\; Fy \;\&\; \neg\Box Fy)$.

The statement (27) is basically a claim that properties can be divided into necessary and contingent. We will show later that (27) is not enough to commit modal logic to claims like "something is necessarily greater than seven". On the other hand, (28) or its paraphrase (29) is more problematic. In addition to previous distinction it says that a property can be such that it is necessary for some object(s) but not for other. This means that the categorization of properties must be more complex than in case of essentialism forced by (27). This version of essentialism is really problematic. Because we can no longer say that it is the property that is necessary, we have to admit, that there is something about the object that "brings about" the necessity. But what is it? It is certainly "the lack, so far, of satisfactory answer to this question […] that makes this version of essentialism a real source of philosophical perplexity."[57] If we now define that a system of logic is committed to essentialism if (i) it has some essential sentence as a theorem or (ii) it requires (presupposes) that some essential sentence be true or (iii) it allows formulation of some essential sentence, then Parsons assures us that modal logic is not forced to accept either version of essentialism. In his argumentation, he gives us two reasons: (i) there are certain, namely maximal Kripkean, models in which no essential sentence is true and (ii) following some basic rules of how to choose axioms of one's system, s/he can always do it in such a way that the system endorses no essential sentence. The rules are: (a) the axioms all be closed and contain no constants and (b) the axioms contain no modal operators except on the front.[58] These two rules then assure that the modal system be not forced to accept essentialism unless some special essentialist axioms are added. Otherwise all essential sentences will be meaningful but false.

Last thing we want to address is Quine's "bewildering" argument with mathematicians and cyclists which we quoted on page 27 . The argument summarized reads as follows

(30) Modalities yield talk of a difference between necessary and contingent attributes.

(31) Mathematicians are said to be necessarily rational and not necessarily two-legged.

(32) Cyclists are necessarily two-legged and not necessarily rational[59]

(33) A is a mathematician and a is a cyclist

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[57] Parsons (1969), p. 38

[58] For detailed argumentation see Parsons (1969), especially section IV and appendix A.

[59] No matter what we think about the argument, this premise is obviously false. In the time of Paraolympics and modern prothetics the two-leggedness (at least in some sense) is no more a prerequisite for cycling therefore certainly not a necessary attribute of cyclists.

The question Quine raises is: Is this concrete individual necessarily rational and contingently two-legged or vice-versa? And he rightly thinks that we shall be bewildered. The problem is that there is probably no interpreted modal system that would allow formulation of such paradox. If we translate (31) - (33) into formal language of some standard modal logic (where 'M' stands for mathematicians, or precise for property to be a mathematician, 'R' for rational beings, 'C' for cyclists and 'T' for two-legged beings), we get:

(34) $\Box(\forall x)(Mx \rightarrow Rx)$

(35) $\Box(\forall x)(Cx \rightarrow Tx)$

(36) Ma & Ca.

But from these three premises we can barely get an answer to Quine's question. Among the results we can get are modalized formulas like $\Box(Ma \rightarrow Ra)$, $\neg\Diamond(Ma \,\&\, \neg Ra)$, $\Diamond(Ma \,\&\, \neg Ta)$, $\neg\Box(Ma \rightarrow Ta)$ and non-modal facts like Ta, Ra, Ta & Ra. Among those results there is nothing that would give answer to Quine. Moreover, there is nothing that would legitimize the question he rises. The way of reasoning Quine would have to invoke is to infer from (34) and (36) the formula

(37) $\Box Ra$.

But that would require inference rule in the form: from $\Box(A \rightarrow B)$ and A infer $\Box B$, which is invalid in any standard system besides the case where A is strictly equivalent to $\Box C$. But there is no such prerequisite in premise (34), so the whole argument is inconclusive. To give Quine another try we might reformulate the premises in de re mode, the first being

(38) $(\forall x)(Mx \rightarrow \Box Rx)$.

Then the inference would be valid and (37) achieved but the premise is now intuitively dubious. Would we normally claim that a person being mathematician in actual possible world is rational in all possible situations. What about the world, where s/he does not exist or where s/he has no brain? So the premise in form of (38) is presumably false and the conclusion of the argument does not follow. Thus we have blocked all possible ways how to make any sense of what Quine utters and we cannot but conclude that he must be wrong here or did not express his idea clearly enough. In any case, bewilderment certainly remains.

To conclude this section let's sum up the result of the debate between Quine and his opponents. We have seen that Quine has lot of points to make and that some of them can be really devastating. We also saw that many of these points are based on certain intuitions about logic and above all on the belief that fully extensional lower predicate calculus is the only real and reliable logic. But sometime Quine claims too much. With the help of his opponents we have shown that there is a moral in his arguments: the combination of modalities and quantification requires certain revision of some part of standard logic. Nevertheless, we have also shown that this revision, in our case the distinction between reference and extension is more than natural. So after all, if we cut off the exaggerated parts of Quine's argumentation, we must admit that there is a sound core, but that it is not sufficient to eliminate modal logic for good. There are certain things a modal logician has to be aware of, but there is always a way of how to build a soundly interpreted system of modal logic with modal operators applied even to open sentences.

3. Modal logic - a formal sketch

3.1. Calculi of Modal Logic

So far we have talked about modal logic without specifying any of its particular features. In this section we would like to introduce a formal characterization of some modal systems. Because there are many systems of modal logic we will limit ourselves on three that are of some philosophical interest with regards to study of alethic modalities. As a result we will set aside all systems that formalize deontic and other modalities. We also consider that our system contain no constants nor complex singular terms (definite descriptions). It is a simplification but it is inessential for demonstration of important features of modal systems.

Let's first define the language L for our modal systems.[60] This will consist of a list of primitive symbols together with definition of formulae. We take as primitive or undefined symbols of L the following:

a) a set of individual variables: x, y, z, … (eventually x_1, x_2, x_3 …),
b) a list of predicates ($n \geq 1$): P^n, Q^n, R^n, … (eventually $P^n{}_1$, $P^n{}_2$, $P^n{}_2$, …),
c) the symbols for negation '$\neg$', disjunction '$\vee$', universal quantifier '$\forall$' , necessity '$\Box$' and auxiliary symbols '(' and ')'.

(We will also use φ, χ, ψ as meta-logical variables for formulas and bold **a**, **b** – with or without numeric index - for individual variables.) Following formation rules now define what is a well-formed formula (wff) or simply a formula of our language:

a) An expression $P^n(x_{1,\dots,}x_n)$ where P^n is a n-adic predicate (also called predicate of degree n) and where $x_{1,\dots,}x_n$ are individual variables is a well-formed formula (wff). We will call such a formula atomic formula.
b) If φ is a wff, then $\neg\varphi$ is a wff.
c) If φ and ψ are wffs, then $(\varphi \vee \psi)$ is a wff provided that any predicate variable or constant, which occurs more than once in $(\varphi \vee \psi)$ is of the same degree on each occurrence.
d) If φ is a wff and **a** is any individual variable, then $(\forall\mathbf{a})\varphi$ is a wff.
e) If φ is a wff, then $\Box\varphi$ is a wff.

Concerning formation rule d), φ in $(\forall\mathbf{a})\varphi$ is so-called scope of the quantifier. Occurrence of **a** in φ is said to be free if it does not lie within the scope of any quantifier in φ which contains **a**. Otherwise it is said to be bound. Further we will define conjunction '&', material implication '$\rightarrow$', material equivalence '≡', existential quantifier '∃', monadic possibility operator '◊' and dyadic strict implication operator '⥽'. The definitions are:

a) $(\varphi\&\psi) =_{df} \neg(\neg\varphi\vee\neg\psi)$,
b) $(\varphi\rightarrow\psi) =_{df} (\neg\varphi\vee\psi)$,
c) $(\varphi\equiv\psi) =_{df} ((\varphi\rightarrow\psi)\&(\psi\rightarrow\varphi))$,
d) $(\exists \mathbf{a})\varphi =_{df} \neg(\forall \mathbf{a})\neg\varphi$,
e) $\Diamond\varphi =_{df} \neg\Box\neg\varphi$,
f) $(\varphi ⥽ \psi) =_{df} \Box(\varphi\rightarrow\psi)$.

Now we can almost put down axioms of our first modal system. It is, nevertheless, a natural requirement that a system of modal should incorporate all non-modal truths. Therefore it might be wise to introduce first the non-modal basis that will be common to our systems – the axioms of lower predicate calculus (LPC). As axioms we will count substitutional instances of following axiom-schemata:

(A1) $(\varphi\vee\varphi)\rightarrow\varphi$

(A2) $\psi\rightarrow(\varphi\vee\psi)$

(A3) $(\varphi\vee\psi)\rightarrow(\psi\vee\varphi)$

(A4) $(\psi\rightarrow\chi)\rightarrow((\varphi\vee\psi)\rightarrow(\varphi\vee\chi))$

(∀1) If **a** is any individual variable, φ a wff and ψ and wff differing from φ only in having some individual variable **b** replacing every occurrence of **a** in φ, then

$(\forall a)\varphi\rightarrow\psi$

is an axiom provided that **a** does not occur within the scope of any occurrence of a quantifier containing **b**.

We further include following transformation rules:

(MP) Modus Ponens: If φ and (φ→ψ) are theses, so is ψ ($\vdash\varphi,\ \vdash(\varphi\rightarrow\psi) \Rightarrow\ \vdash\psi$)

(∀2) If **a** is any individual variable and φ and ψ are any wffs, then

$\vdash\varphi\rightarrow\psi \Rightarrow\ \vdash\varphi\rightarrow(\forall \mathbf{a})\psi$

provided that **a** does not occur free in φ.[61]

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[60] In what follows we have use into a great extent the material found Hughes (1968). In particular we have used chapters 2, 3, 4 (propositional modal logic) and 8 (modal predicate logic).

[61] Several remarks to our axiomatization of LPC. Instead of listed axiom-schemata we could of course take some complete set of axioms plus a rule of uniform substitution. The result would be exactly the same system. Further (∀2) could be replaced by either a combination of axiom-schema $(\forall \mathbf{a})(\varphi\rightarrow\psi)\rightarrow(\varphi\rightarrow(\forall \mathbf{a})\psi)$ (provided **a** is not free in φ) and a rule $\vdash\varphi \Rightarrow\ \vdash(\forall a)\varphi$ (so called universal generalization) or by the same rule and two axiom-schemata $\varphi\equiv(\forall a)\varphi$ (provided **a** is not free in φ) and $(\forall \mathbf{a})(\varphi\rightarrow\psi)\rightarrow((\forall \mathbf{a})\varphi\rightarrow(\forall \mathbf{a})\psi)$. Again the bases would be equivalent to the one with (∀2). For details see Huges (1968) footnotes 79,80.

Now we are finally in a position to introduce the simplest of our modal systems. We will call it LPC+T or simply T. It can be obtained by adding following to the above-mentioned non-modal basis for LPC:

(A5) $\Box\varphi\rightarrow\varphi$ (sometimes called T)

(A6) $\Box(\varphi\rightarrow\psi)\rightarrow(\Box\varphi\rightarrow\Box\psi)$ (sometimes called K)

(N) Rule of Necessitation: $\vdash\varphi \Rightarrow \vdash\Box\varphi$

T itself is a very weak system. It is actually the weakest system that fulfils following intuitive requirements for formalization of our modal reasoning (concerning truth and falsity of statements). These requirements are following. (1) We require that necessity and possibility are somehow interconnected. This leads to the requirement that equivalences $\Diamond\varphi =_{df} \neg\Box\neg\varphi$ and $\Box\varphi =_{df} \neg\Diamond\neg\varphi$ should be valid. It also reduces the amount of primitive symbols of the respective modal system. (2) Strict implication is wished to be interpreted as entailment or a relation closely related to it. The broadly accepted idea is that both entailment and strict implication should respect the condition $(\varphi\dashv\psi)\rightarrow\neg\Diamond(\varphi\&\neg\psi)$. There is, however, some controversy about the converse. No matter what some people think about entailment, modal logicians decided to formalize a relation (maybe identical with entailment) that is defined by $(\varphi\dashv\psi)\equiv\neg\Diamond(\varphi\&\neg\psi)$ and therefore holds between those formulae when and only when it is impossible for φ to be true and ψ to be false. Therefore the above-mentioned equivalence should hold in all standard modal systems. (3) Modal operators are not truth-functional. As a result '$\Box\varphi$', '$\Diamond\varphi$' or '$\varphi\dashv\psi$' cannot be equivalent to any truth-function of φ (respectively φ and ψ). Let's take the case of '$\Box$'. There are exactly four distinct truth-functions of one proposition. If we call it φ, we get φ itself, the negation of φ and further a function that is always True and another one that result of which is always False. And so we get four equivalencies that must not hold in any sound modal system: $\Box\varphi\equiv\varphi$, $\Box\varphi\equiv\neg\varphi$, $\Box\varphi\equiv(\varphi \,\&\, \neg\varphi)$ and $\Box\varphi\equiv(\varphi \vee \neg\varphi)$. (4) However, we require as valid the implication $\Box\varphi\rightarrow\varphi$ and $\varphi\rightarrow\Diamond\varphi$, which cover the commonsense intuitions that what is necessarily true is also true and what is true is also possible true. (5) Further requirement concerns valid formulae (formulae true under all value-assignments). If φ is a valid formula, then we expect as valid not only any formula with the form of φ but also any formula that has the form of $\Box\varphi$ (and of course $\Box\varphi$ itself). (6) Last intuitive principle is that whatever follows from necessary truth is also necessarily true. It means that if we have derived some conclusion from necessary premises, it should not be the case that the conclusion possesses "less truth" than the premises themselves. As a result we can expect $(\Box\varphi\&(\varphi\dashv\psi))\rightarrow\Box\varphi$ or $\Box(\varphi\rightarrow\psi)\rightarrow(\Box\varphi\rightarrow\Box\psi)$ to be a theorem (if not axiom) of sound modal systems.

As we can see T fulfills all of our intuitive criteria but it is itself very weak. It is actually the weakest system that fulfills them all. One of problems with T is the irreducibility of "modalities".

To explain this better let us define modality as any (potentially empty) unbroken sequence of monadic operators (¬, □, ◊). We can further say that two modalities m_1 and m_2 are equivalent in a given system if and only if (further iff) the result of replacing m_1 by m_2 or vice versa in any formula is always equivalent in that system to the original formula. Otherwise we call them distinct. If moreover m_1 contains fewer modal operators than m_2 we say that m_2 is reducible to m_1. If we think of sequences of two modal operators we can get following reduction possibilities: (i) $\Box\varphi \equiv \Diamond\Box\varphi$, (ii) $\Diamond\varphi \equiv \Box\Diamond\varphi$, (iii) $\Box\varphi \equiv \Box\Box\varphi$, (iv) $\Diamond\varphi \equiv \Diamond\Diamond\varphi$. T unfortunately does not include any of reduction rules we mentioned and does not allow for any reduction of complex modalities (in formulas like $\Box\Box\Diamond\Box\Diamond\Box\varphi$ or $\Box\Box\Box\Diamond\Diamond\varphi$). Moreover, for any sequence of modal operators, we can construct a longer one that is not equivalent to it. As a result, T contains infinite number of modalities.

Realizing this feature of T we may wish to construct systems that will allow for some reduction of modalities. Fortunately we do not need to add all the reduction rules to our new system. One "implication half" of every rule is a substitution instance of $\Box\varphi \rightarrow \varphi$ - an axiom of T - or its theorem $\varphi \rightarrow \Diamond\varphi$. The "implication halves" are: $(\Box\varphi) \rightarrow \Diamond(\Box\varphi)$, $\Box(\Diamond\varphi) \rightarrow (\Diamond\varphi)$, $\Box(\Box\varphi) \rightarrow (\Box\varphi)$, $(\Diamond\varphi) \rightarrow \Diamond(\Diamond\varphi))$. From the rest, the pairs $\Diamond\varphi \rightarrow \Box\Diamond\varphi$ and $\Diamond\Box\varphi \rightarrow \Box\varphi$ and $\Box\varphi \rightarrow \Box\Box\varphi$ and $\Diamond\Diamond\varphi \rightarrow \Diamond\varphi$ are derivable from each other. So in the end there are only two non-equivalent reduction rules ($\Box\varphi \rightarrow \Box\Box\varphi$ and $\Diamond\varphi \rightarrow \Box\Diamond\varphi$ where moreover the prior can be derived from the latter but not the other way round) which suggest creation of two systems stronger than T.[62]

The first stronger system including T is created by adding axiom

(A7) $\Box\varphi \rightarrow \Box\Box\varphi$ (sometimes called 4)

to the axiomatic basis of T. It is called S4. The other one, stronger than both S4 and T, is created by addition of

(A8) $\Diamond\varphi \rightarrow \Box\Diamond\varphi$ (sometimes called 5)

to the axioms of T. The transformation rules are the same as in T. Thanks to the reduction possibilities S4 has only 14 irreducible modalities (-[63], □, ◊, □◊, ◊□, □◊□, ◊□◊ and its negations) and allows us (i) to replace any unbroken sequences of □'s or ◊'s by a single □ or ◊ and (ii) if a sequence is more than three operators long to delete all but last three. S5 on the other hand contains only 6 modalities (-, □, ◊ and its negations) and allows us to delete from any sequence of monadic modal operators all but the last one.

There might be however one more reason to adopt S4 or S5 rather than T. It concerns some philosophical views about propositions. S4 can be understood as expressing the fact that all necessary propositions are necessarily necessary. S5 extends this idea so that if a proposition has

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[62] For details about reduction rules (including derivations) see Huges (1968), pp. 44 – 47.

any modal characteristics it has it by necessity. This may be interesting not only for logicians but also for some philosophers. Interesting is that thanks to the fact that S5 is stronger than S4 one can hold the first view independently on the other.

3.2. Semantics

Now when we have introduced the axioms of our modal systems T, S4 and S5 we can move on to their models and defining validity for their formulae.[64] Let's start with a model for T. Such a model is an ordered quadruple $<W, R, D, V>$ where W is a set of objects call them "worlds", R is a reflexive dyadic relation over elements of W ($R \subseteq WxW$), D is a set or "domain" of individuals and V is value-assignment. Before we move on let us to say more concerning the elements of the model. As we said W is a set of objects of some kind for which we will use symbols w_1, w_2, …, w_i, …. We require that it is not empty. From logical point of view we do not have to care what these objects are. It is however part of tradition to call them "worlds" or "possible worlds". This will become clearer in the Chapter 4. R is a relation on a Cartesian product of W such that for every w_i and for every w_j either Rw_iw_j or $\neg Rw_iw_j$. It is often called "accessibility relation". In a sense it defines for every world a set of worlds that are "accessible" from it and thus represent "possible alternatives" to it. For T this relation has to be reflexive, otherwise there are no further criteria. D is a non-empty set of individuals. For the type of model we are presenting right now, D is not relativized to worlds in any sense. There is the same domain of individuals for all elements of W. The value-assignment V is then defined as follows:

a) If x is **a** an individual variable, then V assigns to **a** some element u of D. We write $V(x) = u$.
b) If P is a n-adic predicate, then V assigns to P a set of ordered n+1-tuples of the form $<u_1,\ldots,u_n,w_i>$, where $u_1,\ldots,u_n$ are elements of D and w_i belongs to W. Formally $V(P) = \{<u_1,\ldots,u_n,w_i>; u_1,\ldots,u_n \in D, wi \in W\}$.
c) [Atomic formulae] If P is any n-adic predicate then $V(P(x_1,\ldots,x_n),w_i) = T$ if $<V(x_1),\ldots,V(x_n),w_i> \in V(P)$. Otherwise $V(P(x_1,\ldots,x_n),w_i) = F$.
d) [$\neg$] For any wff φ and any $w_i \in W$, $V(\neg\varphi,w_i) = T$ if $V(\varphi,w_i) = F$. Otherwise $V(\neg\varphi,w_i) = F$.
e) [$\vee$] For any wffs φ and ψ and any $w_i \in W$, $V(\varphi\vee\psi,w_i) = T$ if either $V(\varphi,w_i) = T$ or $V(\psi,w_i) = T$. Otherwise $V(\varphi\vee\psi,w_i) = F$.

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[63] '-' marks blank modality, i.e. a zero long sequence composed of modal operators and negations. Example of formula with zero long modality is Px & Qx or $\forall xPx \rightarrow Py$.
[64] Our guide will be again Huges (1968), Ch. 8. There the reader can find all the details including some derivations and proofs.

f) [∀] For any wff φ, any individual variable **a** and any $w_i \in W$, $V((\forall \mathbf{a})\varphi, w_i) = T$ if for every assignment V' which makes the same assignment as V does to all variables other than **a**, $V'(\varphi, w_i) = T$. Otherwise $V((\forall \mathbf{a})\varphi, w_i) = F$.

g) [□] For any wff φ and any $w_i \in W$, $V(\Box\varphi, w_i) = T$ if for every $w_j \in W$ such that Rw_iw_j, $V(\varphi, wj) = 1$. Otherwise $V(\Box\varphi, w_i) = F$.

We can now define validity for T by saying that a wff φ is T-valid if for every T-model <W, R, D, V>, $V(\varphi, w_i) = T$ for every $w_i \in W$.

The same definition of V will hold for S4 and S5. We only have to make additional requirements that the relation R should be reflexive and transitive for S4 and reflexive, transitive and symmetrical for S5. That, of course, means that R in S5 is an equivalence relation. Therefore in S5 every world is accessible or possible alternative to every other world and so the relation becomes redundant in the definition of model. As a result the model for S5 can be defined only as an ordered triple <W, D, V>. The reference to R can be deleted also from g) of V's definition.

There is however one kind of problem connected with this kind of models. We have mentioned above that the domain of individuals is the same for all possible worlds. The consequence is that the model validates a controversial formula

(BF) $(\forall x)\Box Px \rightarrow \Box (\forall x)Px$.

This formula is called Barcan formula and is also known in its original form

(BF') $\Diamond(\exists x)Px \rightarrow (\exists x)\Diamond Px$.

(Because our above-mentioned model of T respectively S4 and S5 validates (BF), we will call it T+BF-model respectively S4+BF and S5+BF. The same concerns the value assignment and validity.) (BF) has several interesting aspects. Firstly, it is closely related to one feature of the model of modal system, namely with the constant domain of individuals. If (BF) is validated by a model, the model must have constant individual domain and vice versa. Further, in case of T and S4, it is independent on other axioms of the system. In S5, however, it is derivable as a theorem. Because it is a source of many controversies and is rejected by many logicians and philosophers (more about it in the next chapter) many people came up with models that invalidate it. In what follows we will present two types of models that invalidate (BF).

If we deny that the domain of individuals is the same for all possible worlds we have two options. We can admit that in for some w_i there are possibly relative worlds in which there are individuals which are missing in w_i. Or we can be even more liberal and drop any limitations at all. Different worlds have simply different domains. Some individuals can appear, some can be missing. Let's take the less liberal version first. As an example let's take system T again. This time, because BF will come out invalid, we will call it "T minus BF" (T-BF).

We define T-BF model as an ordered quintuple <W, R, D, Q, V> where W, R, D are to be understood as before in T+BF-model. New element is the function Q which assigns to every possible world w_i a non-empty subset D_i of D. We further impose a "inclusion requirement" that for every w_i and w_j such that Rw_iw_j, $D_i \subseteq D_j$. V now differs from the one above in that it does not assign truth-value to every wff but leaves some of them unevaluated. We say that if V assigns some truth-value to φ (in w_i), φ is defined by V (in w_i). Otherwise φ is said to be undefined (in w_i). The value-assignment is now defined as follows:

a) If **a** is an individual variable, then V(**a**) = u.
b) If P is a n-adic predicate, then $V(P) = \{<u_1,\ldots,u_n,w_i>; u_1,\ldots,u_n \in D, wi \in W\}$.
c) If φ is atomic formula $P(x_1,\ldots,x_n)$, then for any $w_i \in W$, if each of $V(x_1),\ldots,V(x_n)$ is in D_i, then $V(P(x_1,\ldots,x_n),w_i)$ = T or F according as $<V(x_1),\ldots,V(x_n),w_i> \in V(P)$ or not. Otherwise $V(P(x_1,\ldots,x_n),w_i)$ is undefined.
d) [¬] For any wff φ and any $w_i \in W$, $V(\neg\varphi,w_i)$ = T iff $V(\varphi,w_i)$ = F and $V(\neg\varphi,w_i)$ = F if $V(\varphi,w_i)$ = T. (If $V(\varphi,w_i)$ is undefined, so is $V(\neg\varphi,w_i)$.)
e) [∨] For any wffs φ and ψ and any $w_i \in W$, $V(\varphi\vee\psi,w_i)$ = T iff both $V(\varphi,w_i)$ and $V(\psi,w_i)$ are defined and either $V(\varphi,w_i)$ = T or $V(\psi,w_i)$ = T and $V(\varphi\vee\psi,w_i)$ = F iff both $V(\varphi,w_i)$ and $V(\psi,w_i)$ are defined and either $V(\varphi,w_i)$ = F or $V(\psi,w_i)$ = F. (If $V(\varphi,w_i)$ or $V(\psi,w_i)$ are undefined so is $V(\varphi\vee\psi,w_i)$.)
f) [∀] For any wff φ, any individual variable **a** and any $w_i \in W$, $V((\forall\mathbf{a})\varphi,w_i)$ = T iff for every V' which assigns to **a** some member of D_i and is otherwise the same as V, $V'(\varphi,w_i)$ = T and $V((\forall\mathbf{a})\varphi,w_i)$ = F iff there is some such V' that $V'(\varphi,w_i)$ = F. (Otherwise $V((\forall\mathbf{a})\varphi,w_i)$ is undefined.)
g) [□] For any wff φ and any $w_i \in W$, $V(\Box\varphi,w_i)$ = T iff for every $w_j \in W$ such that Rw_iw_j, $V(\varphi,w_j)$ = T and $V(\Box\varphi,w_i)$ = F iff for every such w_j $V(\varphi,w_j)$ is defined and for some such w_j $V(\varphi,wj)$ = F. (If for some w_j $V(\varphi,w_j)$ is undefined then $V(\Box\varphi,w_i)$ is also undefined.)

A wff φ is now T-BF-valid if for every T-BF-model <W, R, D, Q, V>, and for every $w_i \in W$, $V(\varphi,w_i)$ = T wherever defined. Analogous definitions can be derived for S4-BF- and S5-BF-models. (S5-BF model can again be defined without mentioning the relation R.) We only have to impose additional criteria on the relation R. We can also easily show that (BF) in now invalid in both T-BF- and S4-BF-model.[65] In S5-BF, (BF) however remains valid due to the fact that in S5 the relation R

[65] The counterexample to (BF) is quite simple. We take $W=\{w_1, w_2\}$, $R=\{<w_1,w_1>, <w_2,w_2>, <w_1,w_2>\}$, $D=\{u_1, u_2\}$. Then we assign D1 = $\{u_1\}$ and D2=$\{u_1,u_2\}$. Further we decide that V(x) be always u_1 and for any other individual variable **a** other than x V(**a**) = u2. When we now take a predicate P = $\{<u_1,w_1>, <u_2,w_2>\}$ we can show that $V((\forall x)\Box\varphi x,w_1)$ = T but $V(\Box(\forall x)\varphi x,w_1)$ = F. Hence (BF) is not valid in this model and we have found a counterexample. For details see Huges (1968), p. 173.

is symmetrical but in our counterexample this requirement is not satisfied. As a result our counterexample is not a valid S5 model.

Last model we want to introduce is the one presented in Kripke (1963)[66]. In Kripke's model there is no restriction on individual domains. For every world there is an independent domain of individuals. That means that Kripke's model covers the intuition that the individual domain can vary from world to world in any respect. Moreover it assigns truth-values to every formula so that we are left with no undefined formulas in the end. Otherwise the model is very similar to our T-BF model above. Kripke's model (in our case for system T) is again an ordered quintuple <W, R, D, Q, V> where W, R, D are to be understood as before. Function Q assigns to every possible world w_i a non-empty subset D_i of D. There are no further requirements on Q. The value-assignment V is now defined as follows:

a) If **a** is an individual variable, then V(**a**) = u, $u \in D$.
b) If P is a n-adic predicate, then $V(P) = \{<u_1,\ldots,u_n,w_i>; u_1,\ldots,u_n \in D, wi \in W\}$ Note that $u_1,\ldots,u_n$ do not necessarily belong to D_i.
c) If φ is atomic formula $P(x_1,\ldots,x_n)$, then for any $w_i \in W$, $V(P(x_1,\ldots,x_n),w_i) = T$ or F according as $<V(x_1),\ldots,V(x_n),w_i> \in V(P)$ or not.
d) [$\neg$] For any wff φ and any $w_i \in W$, $V(\neg\varphi,w_i) = T$ if $V(\varphi,w_i) = F$. Otherwise $V(\neg\varphi,w_i) = T$.
e) [$\vee$] For any wffs φ and ψ and any $w_i \in W$, $V(\varphi\vee\psi,w_i) = T$ if either $V(\varphi,w_i) = T$ or $V(\psi,w_i) = T$. Otherwise $V(\varphi\vee\psi,w_i) = F$.
f) [$\forall$] For any wff φ, any individual variable **a** and any $w_i \in W$, $V((\forall \mathbf{a})\varphi,w_i) = T$ if for every assignment V' which assigns to **a** some member of D_i and is otherwise the same as V, $V'(\varphi,w_i) = T$. Otherwise $V((\forall \mathbf{a})\varphi,w_i) = F$.
g) [$\Box$] For any wff φ and any $w_i \in W$, $V(\Box\varphi,w_i) = T$ if for every $w_j \in W$ such that Rw_iw_j, $V(\varphi,wj) = T$. Otherwise $V(\Box\varphi,w_i) = F$.

We can now define Kripkean style of validity for T (T_K-validity) by saying that a wff φ is T_K-valid if for every Kripkean T-model <W, R, D, Q, V>, $V(\varphi,w_i) = T$ for every $w_i \in W$.

The important change in Kripke's model against the previous models is the interpretation of quantifiers. As we can see in point f) of the value-assignment definition, "everything is P" is now true in w_i if everything in D_i is P. So if there is some $u \in Dj$, $i \neq j$, which is not P, Kripke's semantics will yield $V((\forall x)Px,w_i) = T$ but $V(Py,w_i) = F$. That means that the formula $(\forall x)Px \rightarrow Py$ is not valid and because it is an instance of our axiom-schema ($\forall$1) it means that the Kripkean semantics invalidates the schema as well. Kripke is aware of this and deals with it in a following way.

[66] Our notation and the way of presentation however copies the one of Huges (1968), p. 178-80.

Although $(\forall x)Px \rightarrow Py$ is not valid in his semantics the universal closure of it – $(\forall y)((\forall x)Px \rightarrow Py)$ – is. Similarly his semantics validates the universal closure of any instance of ($\forall$1). It concerns also every theorem derived by ($\forall$2). So Kripke has to do one modification to the non-modal basis of his modal system. He has to use such axiomatization of LPC that yeild as theorems only closed formulae. This is however not that serious because in the standard LPC it holds that a open formula is valid iff its universal closure is valid[67]. Therefore we have not lost anything from the expressive power of LPC. So if we want to use Kripke's semantics for our modal systems we only have to modify their non-modal part.

Kripkean semantics has one important feature. It invalidates (BF) not only in T and S4 but also in S5. It also invalidates even the converse of (BF), the formula

(CBF) $\Box(\forall x)\varphi \rightarrow (\forall x)\Box\varphi$,

which was derivable as a theorem even in T. That makes an important difference between our interpreted systems T+BF, T-BF (S4+BF, S4-BF and S5) and T (S4, S5) interpreted by Kripkean semantics. So the choice of a particular semantics may influence the choice of non-modal axioms of the system and as a result the choice of one or another interpreted modal system for some philosophical or other purpose may have serious consequences regarding for example ontological commitments or ontological views.

3.3. *Identity*

The systems we have seen so far were presented without identity. In what follows we will sketch how things would have to change in order to add identity to T, S4 and S5. In LPC (and modal systems that use LPC as a basis) identity ('=') is a primitive symbol and can be added by introducing two more axioms to the LPC basis of our systems.

(I1) $\mathbf{a}=\mathbf{a}$

(I2) $(\mathbf{a}=\mathbf{b}) \rightarrow (\varphi \rightarrow \psi)$ where φ and ψ differ only in that in one or more places where φ has free **a**, ψ has free **b**.

For simplicity we will present only systems with BF and call T+BF, S4+BF and S5 with addition of I1 and I2 T+I, S4+I and S5+I. There is not much to say about identity in non-modal contexts. As usual it is an equivalence relation. In modal context we, however, get some problem. After addition of (I1) and (I2) we can (even in T+I) derive a counterintuitive formula

(NI) $(x=y) \rightarrow \Box(x=y)$.

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[67] A universal closure can be defined as follows. If φ is a formula with free variables $x_1, \ldots, x_n$ then the formula

In S5 (but not in weaker systems) we can moreover derive a related formula

(NNI) $(x\neq y)\rightarrow\Box(x\neq y)$.

It is natural to think that (NI) and (NNI) should stand and fall together. As we have said, (NNI) is a theorem of S5+I. To T+I and S4+I we have to add it as a new axiom (or axiom-schema). We also would have to add to the formation rules a rule that expression '$x_1=x_2$' is an atomic formula. The value-assignment for systems including (NNI) will have to be extended to cover formulas with identity. The additions are as follows:

b) If P is a n-adic predicate, then V assigns to P a set of ordered n+1-tuples of the form $<u_1,\ldots,u_n,w_i>$, where $u_1,\ldots,u_n$ are elements of D and w_i belongs to W. Formally $V(P) = \{<u_1,\ldots,u_n,w_i>; u_1,\ldots,u_n\in D, wi\in W\}$. Further $V(=) = \{<u, u, w_i>; u\in D, wi\in W\}$

c1) [Atomic formulae] If P is any n-adic predicate, then $V(P(x1,\ldots,xn),wi) = T$ if $<V(x1),\ldots,V(xn),wi>\in V(P)$. Otherwise $V(P(x1,\ldots,xn),wi) = F$.

c2) [=]. For any individual variables a and b and for any $wi\in W$, $V(a=b,wi) = T$ or F according as $V(a) = V(b)$ or not.

Otherwise V is the same as the value-assignment for T+BF model. And similarly for S4+I and S5+I.

There are of course ways to avoid validity of (NI) and (NNI). We will discuss them briefly. First option is to replace the axiom-schema (I2) by a weaker version that reads

(I2') $(\mathbf{a}=\mathbf{b})\rightarrow(\varphi\rightarrow\psi)$

where φ and ψ differ only in that in one or more places where φ has free **a**, not occurring within the scope of modal operator, ψ has free **b**.

Such change would immediately block the derivation of both (NI) and (NNI) in all systems including S5. Consequently we would have make some modification to the definition of value-assignment including the parts for identity, atomic formulae and quantification. As a result we would get another intuitively implausible formula $\Box(\exists x)Px \rightarrow (\exists x)\Box Px$ as valid. Another way to include identity, which we will not elaborate on, would be to build systems (from T and S4) in which (I1) and (I2) but not (NNI) would be theses.[68]

$(\forall x_1)\ldots(\forall x_n)\varphi$ is a universal closure of φ.

[68] For more on both options see Huges (1968), pp. 195 – 202.

3.4. Definite Descriptions

When we were talking about modal logic in Chapter 2 we always talked about systems that include singular terms (names, definite descriptions). Also the main objection to validity of Quine's argument against modal logic rested on the difference between names and definite descriptions. Systems of modal logic presented in this chapter so far however did not include expressions of these categories. All we had were variables. In the last section of this chapter we would like to show one way of how to include singular terms into a modal system. It is quite difficult to find an axiomatic modal system that would explicitly contain definite descriptions and provide an intuitive and unproblematic semantics for them. The main problem is that definite descriptions (e.g. "the round square") may fail to denote any individual. This feature of them forces us to use a non-standard quantificational theory (because standard one does not allow for non-denoting singular terms). Further, many systems that treat definite descriptions in one way or other usually contain assumptions (usually for technical convenience) that are not always acceptable if we think about the modal system as of a tool for analysis of natural language and our modal reasoning. These assumption may include assignment of arbitrary individual to a non-denoting singular terms, assuming that there is an individual constant for every individual (i.e. intuitively that every individual has a name) or defining validity so that any atomic formula with non-denoting terms is false. These assumptions are innocuous from the technical point of view but might be (and are) subject to philosophical disputes. For the presented system we will use some of these assumptions as well.

Our goal is nevertheless to present an example of how singular terms (including those without referent) can be handled in system of modal logic. For our demonstration we will use system S5 with identity and with fixed domain of individuals. Our understanding of logical constants is a standard one. A logical constant denotes one and only one individual. In modal contexts it denotes the same individual throughout all possible worlds.[69] Definite descriptions on the other hand enable us to identify a thing by its unique property. A definite description is usually said to be "an expression of the form 'the so-and-so' where 'the' is in the singular".[70] So 'the highest mountain' or 'my favorite joke' are examples of definite descriptions in natural language. To represent definite descriptions in formal system we add a description operator 'ι'. It functions

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[69] With Kripke we will call such an expression a rigid designator. In general, we will say that an expression is rigid iff it designates the same individual in all possible worlds in which the expression is defined (intuitively in which the individual exists). If moreover, the expression is defined (the individual exists) in all possible worlds, the expression is called strongly rigid.

[70] Lambert (1997), p. 97.

similarly to a quantifier. It binds a variable and together with a wff, say φ, forms a new expression of the form (ɿx)φ. It differs, nevertheless, from quantifier in that the resulting expression is not a wff but a (complex) singular term. Now the language of our system $S5_D$ will have following primitive symbols:

a) a set of individual variables: x, y, z, … (eventually x_1, x_2, x_3 …),
b) a set of individual constants: a, b, c, … (eventually a_1, a_2, a_3 …),
c) a list of predicates (n≥1): P^n, Q^n, R^n, … (eventually P^n_1, P^n_2, P^n_2, …),
d) the symbols for negation '¬', material conditional '→', universal quantifier '∀' , description operator 'ɿ', necessity '□' and auxiliary symbols '(' and ')'.

Our formation rules now simultaneously define both formulas as well as singular terms. (Besides the meta-variables for formulas and variables, we will use s, t – with or without numerical indexes – as meta-variables for terms.) So following are the formation rules of $S5_D$:

a) Every variable is a term.
b) Every individual constant is a term.
c) If P^n is a n-adic predicate, and $t_{1,\dots,}t_n$ are terms, then $P(t_{1,\dots,}t_n)$ is a (atomic) formula
d) If s and t are singular terms, then s=t is a (atomic) wff.
e) If φ is a wff, then ¬φ is a wff.
f) If φ and ψ are wffs, then (φ ∨ ψ) is a wff provided that any predicate variable or constant which occurs more than once in (φ ∨ ψ) is of the same degree on each occurrence.
g) If φ is a wff and **a** is any individual variable, then (∀**a**)φ is a wff.
h) If φ is a wff, then □φ is a wff.
i) If **a** is any variable and φ is a wff, then (ɿ**a**)φ is a (complex) singular term.

We modify the definitions of operators listed in section 3.1 by changing points a) and b) to

a) $(\varphi\vee\psi) =_{df} (\neg\varphi\rightarrow\psi)$,
b) $(\varphi\&\psi) =_{df} \neg(\neg\varphi\vee\neg\psi)$.

Let us now define one notion – modal closure. A modal closure of a formula φ is the result of prefacing any string of '□'s to φ. By convention, modal closures of an axiom-schema are the results of applying any string of '□'s to any instance of the schema.] The axioms of $S5_D$ are now[71]:
modal closures of following axiom-schemata:

(A1) $\varphi\rightarrow(\psi\rightarrow\varphi)$,

[71] It is quite difficult to find a complete axiomatization of modal logic with definite descriptions. The only version we have found was the one in §§ 1-2 of an unpublished draft of Principia Metaphysica of Edward N. Zalta. (The latest version of the this document can be found at URL http://mally.stanford.edu/publications.html.) Our axiomatization of $S5_D$ is a very slightly modified copy of it.

(A2) $(\varphi\rightarrow(\psi\rightarrow\chi))\rightarrow((\varphi\rightarrow\psi)\rightarrow(\varphi\rightarrow\chi))$,

(A3) $(\neg\varphi\rightarrow\neg\psi)\rightarrow((\neg\varphi\rightarrow\psi)\rightarrow\varphi)$,

(A4) $\forall\mathbf{a}\varphi\rightarrow((\exists\mathbf{b})\mathbf{b}=t\rightarrow\varphi(t/\mathbf{a}))$ where t is substituable for **a,**

(A5) $(\exists\mathbf{b})\mathbf{b}=t$ for any t free of definite descriptions,

(A6) $\psi(t/\mathbf{a})\rightarrow(\exists\mathbf{b})\mathbf{b}=t$, where t is any term containing a description and $\psi(t/\mathbf{a})$ is an atomic formula in which t has been substituted for the variable **a,**

(A7) $(\forall\mathbf{a})(\varphi\rightarrow\psi)\rightarrow(\varphi\rightarrow(\forall\mathbf{a})\psi)$, where **a** is not free in φ,

(A8) $(\forall\mathbf{a})\mathbf{a}=\mathbf{a}$,

(A9) $a=b\rightarrow[\varphi\rightarrow\varphi(a//b)]$,

(A10) $\Box(\varphi\rightarrow\psi)\rightarrow(\Box\varphi\rightarrow\Box\psi)$,

(A11) $\Box\varphi\rightarrow\varphi$,

(A12) $\Diamond\varphi\rightarrow\Box\Diamond\varphi$,

(A13) $\forall a\Box\varphi\rightarrow\Box\forall a\varphi$.

Further we will have following rules:

(MP) $\vdash\varphi\rightarrow\psi,\varphi\Rightarrow\psi$

(GEN) $\vdash\varphi\Rightarrow\ \vdash(\forall\mathbf{a})\varphi$

To introduce Russellian analysis of definite descriptions we let instances (but not modal closures) of following schema to be axioms of the system:

(A14) $\psi(\iota x\varphi/z)\equiv(\exists x)(\varphi\ \&\ (\forall y)(\varphi(y/x)\rightarrow x=y)\ \&\ \psi(x/z))$.

From the derived rules the most important are:

(UI) Universal Instantiation: if τ is free of descriptions, then $\vdash\forall\mathbf{a}\varphi\Rightarrow\ \vdash\varphi(\tau/\mathbf{a})$

(EI) Existential Generalisation: $\vdash\varphi(t/a)\Rightarrow\ \vdash(\exists a)\varphi$, where t is free of descriptions and substituable for a and $\vdash\varphi(t/a),(\exists b)b=t\Rightarrow\ \vdash(\exists a)\varphi$, provided t is any term substituable for **a.**

The Rule of Necessitation (RN) is also available. We have to define first one auxiliary notion - dependence. Suppose that $\varphi_1, \ldots, \varphi_n=\varphi$ is a proof of φ and that ψ is in Γ. Then we say that (the proof of) φ_i $(1\leq i\leq n)$ depends upon the formula ψ iff either (i) φ_i is ψ and the justification for φ_i is that it is a logical axiom or in Γ, or (ii) φ_i follows by (MP) or (GEN) from some previous members of the sequence at least one of which depends upon ψ. Let us now say that a modal axiom is any axiom other than an instance of (A14). Let us also say that a formula ψ is modal with respect to the set Γ iff the formula $\Box\psi$ is in Γ. Then:

(RN) If $\Gamma\vdash\varphi$ such that φ depends only on modal axioms and formulas modal with respect to Γ, then $\Gamma\vdash\Box\varphi$.

Before we can move on, several explanations are needed. Schemata (A1) – (A3) together with (MP) guarantee that we get all propositional tautologies as axioms. Our quantificational theory is expressed by (A3) – (A7) together with (GEN). Because we have now non-denoting terms in the system the quantificational theory of constants (and referential singular terms in general) is a standard one (see also rules (UI) and (EI)) but the logic of definite descriptions is free of existential import.[72] This fact is expressed by (A3) – (A5). Axioms (A8) and (A9) state classical logic of identity, (A10) - (A12) are modal axioms K, T, 5 and we also include BF among axioms of our system (A13). (A14) further symbolizes the analysis of definite descriptions.

The model for $S5_D$ will be again a triple <W,D,V> (because R is equivalence it can be omitted from the definition). W and D are to be understood as before. V is again a value-assignment only that this time it is a partial function. Few remarks to our semantics. Definite descriptions will be interpreted as rigid. That means that if fulfilled they will denote the only one individual that fulfills them in the actual world. If there is no unique individual that denotes them, they will be undefined. Further any atomic formula including undefined terms will be false.[73] So the value-assignment V is defined as follows:

aa) If t is a simple term, then V assigns to t some element u of D. We write V(t) = u.

ab) If t a complex term (definite description $\iota x\varphi$), then if there is $d \in D$ such that for any simple term s, $V(\varphi(s/x)) = T$ iff $V(s) = d$, then $V(\iota x\varphi) = d$; otherwise $V(\iota x\varphi)$ is undefined.

b) If P is a n-adic predicate, then V assigns to P a set of ordered n+1-tuples of the form $<u_1,\ldots,u_n,w_i>$, where $u_1,\ldots,u_n$ are elements of D and wi belongs to W. Formally $V(P) = \{<u_1,\ldots,u_n,w_i>; u_1,\ldots,u_n \in D, wi \in W\}$. Further $V(=) = \{<u, u, w_i>; u \in D, wi \in W\}$

ca) [Atomic formulae]. If P is any n-adic predicate and $t_1,\ldots,t_n$ are terms, then $V(P(t_1,\ldots,t_n),w_i) = 1$ if $V(t_1),\ldots,V(t_n)$ are all defined and $<V(t_1),\ldots,V(t_n),w_i> \in V(P)$. Otherwise $V(P(t_1,\ldots,t_n),w_i) = 0$.

cb) [=] For any terms t and s and for any $wi \in W$, $V(t=s,w_i) = T$ if V(t) and V(s) are defined and V(t) is the same as V(s). Otherwise $V(t=s,w_i) = F$.

d) [¬] For any wff φ and any $w_i \in W$, $V(\neg\varphi,w_i) = T$ if $V(\varphi,w_i) = F$. Otherwise $V(\neg\varphi,w_i) = F$.

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[72] That means that unlike in standard LPC we do not require that definite descriptions have a reference. As a result we have to modify a big part of standard quatificational theory to make sure that terms without reference cannot be substituted into universal claims etc. For more on free logic see Lambert (1997). A modal system S5 with free quantificational part can be found in Fine (1978).

[73] These two assumptions are of course unintuitive. We make them above all for simplicity of presentation. They both can be dropped. If we drop the rigidity criterion, then we have to drop the extensional model of modal system in question. In intensional model, a definite description is (usually) understood as a function from worlds to individuals. As such it can assign different individuals to different possible worlds. Concerning the falsity of atomic formulae with undefined terms we can of course come up with a system that yields at least some atomic formulae (i.e. those with identity) with undefined terms true. An example of non-modal basis for such a system can be found in Lambert (1997), p. 101 ff.

e) [$\vee$] For any wffs φ and ψ and any $w_i \in W$, $V(\varphi \vee \psi, w_i) = T$ if either $V(\varphi, w_i) = T$ or $V(\psi, w_i) = T$. Otherwise $V(\varphi \vee \psi, w_i) = F$.
f) [$\forall$] For any wff φ, any individual variable **a** and any $w_i \in W$, $V((\forall \mathbf{a})\varphi, w_i) = T$ if for every assignment V' which makes the same assignment as V does to all variables other than **a**, $V'(\varphi, w_i) = T$. Otherwise $V((\forall \mathbf{a})\varphi, w_i) = F$.
g) [$\Box$] For any wff φ and any $w_i \in W$, $V(\Box\varphi, w_i) = T$ if for every $w_j \in W$ such that Rw_iw_j, $V(\varphi, wj) = T$. Otherwise $V(\Box\varphi, w_i) = F$.

$S5_D$ validity can be now defined as before. A wff φ is $S5_D$-valid if for every $S5_D$-model <W, D, V>, $V(\varphi, w_I) = T$ for every $w_i \in W$.

With $S5_D$ we will conclude our list of modal systems. We have seen that there are many ways and options how to construct a system of modal logic. This includes above all (i) the choice of modal axioms (ii) the choice of one fixed domain of individuals or varying domains for different worlds (with or without inclusion requirement), (iii) letting the quantifiers range over all individuals in model or only individuals "existent" in a given possible world, (iv) allowing for singular terms and especially non-denoting singular terms (e.g. unfulfilled definite descriptions) and many others. All these options together represent a huge spectrum of variants and from logical point of view they make very little difference. But for a philosopher and metaphysician a choice of particular system may have and has serious impacts on their philosophical views and their ontology. And this idea brings us to the next chapter.

4. Philosophical Interpretation of Modal Logic

4.1. From Logic To Ontology

Any system of modal logic can be looked upon from several different levels. In previous chapter we have presented the formal view. A system of modal logic was defined by its syntax (language, formation rules for terms and formulas, axioms etc.) and formal semantics (model). With these two notions we can do lot of things. We can define validity for formulae or prove completeness or consistency of the system. Is the formal syntax and formal semantics, however, enough to say that the system in question is a system of modal logic? We do not think so.

Let us together with Susan Haack[74] distinguish four levels of a system of logic:

(i) axioms and rules of inference,
(ii) the formal interpretation of (i),
(iii) the informal interpretation (the ordinary language reading) of (i),
(iv) the informal explanation or account of (ii).

Level (i) is the level of syntax and together with level (ii) form a formal system. In such a system we can produce formulae, derive some formulae from other etc. But how can we tell that such a system is a system of for example modal logic? We only have formal symbols and some formal structures - mathematical models. Remember for example the model for our system S4+BF. The only requirement was that it should be an ordered quadruple <W, R, D, V> where W is a set, D is a set R is a relation on members of W and V is a function that assigns T or F to the wffs of the system. But such an interpreted system can be a system of virtually anything. It can be for example a model for electrical circuits, where 'T' and 'F' stand for 'on' and 'off'. So why should it count as a system of modal logic? To understand S4+BF as a system of modal logic we need levels (iii) and (iv). Only they tell us that 'T' and 'F' is to be interpreted as 'true' and 'false' and that logical symbols such as '&' or '→' mean sentence operators 'and' and 'if-then'. Levels (iii) and (iv) give us the intended reading of the calculus and of course its intended interpretation. They also restrict, which ordered quadruples will count as models and how shall we understand them. And last but not least, it is (iv) that determines the commitment to particular ontological and philosophical views. Let us now call level (ii) pure and (iv) depraved semantics.[75] We have already said that it is the depraved semantics that makes our system the system of modal logic. Why? Because it says that wff of our system will represent sentences, 'T' and 'F' truth-values and further that only such

[74] Haack (1978), Chapters 3.2 and 10.4.
[75] A distinction taken from Plantinga (1982), p. 126–8.

quadruples <W, R, D, V> will count as models where W is a set of possible worlds (e.g. sets of propositions or states of affaires), Rw_iw_j means that w_j is possible alternative to w_i and D is set of individuals (people, animals, things, maybe also events etc.). Our decision what to include in W or D also determine our philosophy and ontology. If D can include non-existing objects (Pegasus) then we are forced to include them into our ontology. If we interpret W as a set of maximal states of affairs then our ontology must include states of affairs.

This brings us to two important questions about depraved semantics: Do we need it and if yes how seriously do we have to take it? The first part of the question should be answered in positive. We have tried to show that without informal account of formal semantics we cannot claim that some calculus is a system of something concrete – in our case necessity and possibility. Someone might oppose and say that logic consists in nothing but uninterpreted formalism. Well why not. But if logic is nothing but formalism then you cannot use it for analysis of natural language and philosophical arguments. If you want to do so, you have to find a suitable depraved semantics that will connect the formal system (a set-theoretical construction) with the intuitive notions of truth and falsity (and in our case also necessity and possibility).

The second part of the question is more problematic. One view is that the whole talk of possible worlds is nothing more than a heuristic device and that it should not be taken seriously. We can talk about them but it does not commit us to acknowledging their existence. There are however influential philosophers who thought that it is not that simple. A classical example is Plantinga on possible worlds and D. Lewis' counterpart theory: "We can take it [the counterpart theory] as a heuristic device, an aid to the imagination whose metaphysical imagery makes for vividness but is not to be taken seriously; but we can also take it as an attempt to spell out the sober metaphysical truth about modality. A semantical system […] can be looked at in these same two ways. We may regard its talk of possible worlds and sets of individuals as convenient but dispensable imagery whose cash value is to be found in the insights provided into the workings of language. And if we do look on these semantical systems in this light, then we need not be troubled by embarrassing metaphysical questions about the nature of possible worlds and the status of objects that, as we picturesquely put it, exist only in other possible worlds. […] This attitude towards the semantics, however, is an extremely sophisticated one that does not always stop short of sophistry."[76] We would like to join Plantinga and claim that the depraved semantics of modal logic has to be taken seriously. Let us give some motivations for that.

First of all a system of modal logic that intends to be more than a result of purely formal logical game of producing further and further systems by adding and withdrawing various axioms is

[76] Plantinga (1982), p. 125.

a formalization of something - usually of some pre-theoretic notions. In our case it is the notion of necessity (and possibility). A classical system of modal logic explains the fact that proposition p is necessary in terms of p's being true in all possible worlds and thus elucidates the notion in question. Suppose we are now interested in knowing whether certain proposition is necessary or whether certain philosophical argument with modal notions is valid. Given some system of modal logic L we can translate the argument or the proposition into the formal language of L, analyze it and see whether the proposition is true in all possible worlds or whether the conclusion in the argument follows logically from premises. If not, we can use the semantics to build counterexamples. Suppose we also say that there are in fact no possible worlds. Let's take some argument and suppose it is valid. If there are no possible worlds, then we have maybe proved that our argument is valid in some model. But validity in a model (similarly to truth in a model) is something else than validity simpliciter. We, however, want the argument to be valid in the intended model – in reality. So if the analysis is to be adequate our world must be correctly represented by the formal model. And if there are possible worlds in the formal model, they must have counterparts in reality. If it is not the case, then the fact that the argument is valid in some model has nothing to do with our initial question, which was whether the argument was valid simpliciter. Plantinga in (1982) has similar worries. He imagines a case when we decide to explain (with the help of semantics) the difference between essential and accidental properties. "We may […] propose to explain 'Johan is essentially a person but contingently a philosopher' as the assertion that Johan has both these properties and has the former but not the latter in every world he graces. […] But suppose we refuse to say that there really are any possible worlds or disclaim any views as to what they might be like, or reject any responsibility for the assertion that Johan has properties in some that he lacks in others: then it requires a well-trained eye to see just what our explanation accomplishes."[77] Second motivation comes from classical non-modal logic. There we also have to use depraved semantics to make sure that a system of logic is a sentential logic and not something else. But when we interpret 'T' and 'F' as 'true' and 'false' we also take this interpretation literally and not as a heuristic device that sheds more light on truth and falsity.

Another important issue linked to possible world semantics is the one of epistemological independence. D. Lewis in (1986) and elsewhere criticizes that competing explanations of possible worlds have at least one of the modal notions as primitive (while his own supposedly does not). For example a possible world as a maximal consistent set of sentences uses the notion of consistency. But (semantic) consistency is usually defined in terms of possibility. Therefore such account does not according to Lewis describe possibility independently enough. We can ask whether it is all right

[77] Ibid., p 125-6.

to require such independence. But Lewis requires even more. He requires that the account of pure semantics on the level of depraved one should be non-circular, explanatory and informal. Accordng to Haack Lewis requires that the depraved semantics should be given "in terms which are, so to speak, epistemologically independent of the readings of the modal operators, that one should be able to tell whether there is a possible world in which A independently of one's beliefs about whether possibly A."[78] The question is whether this is possible at all. Haack also points out that even Lewis' own theory fails to match this requirement. So maybe it is a feature of all theories of modality that take depraved semantics seriously. Let us consider following situation. We have a pre-theoretic notion of necessity and we want to formalize it. We sesign a formal system that includes the operator 'Nec'. This operator is then given an informal reading 'it is necessary that'. When we now produce formal semantics for our formal system with 'Nec' we want to supply it with depraved semantics as well (to assure that it is in fact system that deals with necessity and not with something else). But if our informal account is close to the reading, we will naturally violate Lewis' requirements. If it is not, we will see it as inadequate.[79] So it might be "too much to ask that neither 'necessary' nor 'possible' nor any equivalents thereof appear in the patter [= informal account]; explanations of meaning must end somewhere. This isn't to say that […] there is no point in giving patter which elaborates on the original readings; one can after all be helped to understand something by being told the same thing another way."[80]

Before we make the last step towards more philosophical issues and abandon logic there is one last comment to be made. We have seen that purely logical approach to modalities is not appropriate one. If we want to produce a system of modal logic that formalizes the notions of necessity and possibility as we know them, we have to provide it with depraved semantics. That is task mainly for philosophy and especially metaphysics. In logic, a typical approach is to collect some axioms, provide it with some model-theoretic semantics, prove that the new system is consistent (eventually complete), investigate into its formal features and finally determine its positions among other systems of given type. But if we want to produce a system of modal logic we cannot make do without depraved semantics. We, of course, do not have to take possible worlds as "real" worlds à la D. Lewis. We can come up with some abstract surrogates but we cannot deny that there is something in reality that at least represents the formal possible worlds. This idea takes us definitely from logic to philosophy and also to the next section.

[78] Haack (1978), p. 190.
[79] Haack's example (Ibid., p.190) of such inadequacy is to say that p is a possible world iff it is a country in the southern hemisphere.
[80] Ibid.

4.2. Towards Possible Worlds

4.2.1. Unsuitable Conceptions of Possible Worlds

Our first metaphysical question is the one of origin and nature of possible worlds. Since the boom of possible world semantics in 1960's philosophers and logicians have come with many proposals ranging from extreme realism about possible world to proposals how to make do without them. These systems are also based on different intuitions and so the evaluation of them is very difficult. Maybe if we first establish some criteria and formulate some intuitions concerning the nature of reality, we will be able to discriminate proposals that are better than the others and maybe even find the preferred solution. It might be nevertheless use to start with short survey of basic types of possible world semantics.

On the field of possible world semantics we can distinguish four or five main streams which differ as to how seriously do they take the notion of possible world (or in the words of previous section – how seriously do they take depraved semantics): extreme modal realism of D. Lewis, modal realism, modal reductionism, modal fictionalism and finally eliminativism. The first three take the idea of possible world seriously and the proponents came up with explanations of what possible worlds are (real spatiotemporal universes like our world, sets of intensional entities or some abstract constructs). The other two are characteristic by trying to avoid any commitment to possible worlds (while enjoying the benefits of possible-world semantics).

We will start our survey with brief introduction of modal fictionalism and modal eliminativism.[81] As we will see modal realism is committed to existence of possible worlds of some kind and thus open to various objections. Modal fictionalism avoids the commitment to possible worlds by translating modal sentences of the form 'it is possible that p' not into the usual form 'there is a possible world in which p is true' but into

(1) According to modal realism there is a possible world in which it is true that p

Such paraphrase certainly avoids any commitment to possible worlds but is naturally open to two objections. First of all, does this translation capture the ordinary meaning of modalized statements and secondly, can we really still enjoy the benefits of possible-world semantics? According to us, both are to be answered in negative. Our modal statements are usually understood as being about how things are, how they could be and so on. They are certainly not intended to be about how certain theory treats modalities. Moreover, there are various versions of modal realism that treat modalities, possible words and individuals in different ways. Which one shall then enter into (1)? Concerning the second point one of the main benefits of possible world semantics is that modal statements and arguments containing them can be evaluated through a direct application of ordinary extensional predicate calculus. But is it of any use, if we do not take the translation of modal statements into the language of possible worlds as preserving their meaning?

Modal eliminativism on the other hand is the view that "statements that appear to talk about possible worlds, or possible individuals, do not in fact do so: all expressions that appear to refer to possible worlds, or possible individuals, are non-referring expressions."[82] As a result it is a mistake to invoke any kind of possible worlds and the truth conditions have to be formulated without them. Such theory can be characterized by one of the following definitions:

(2) It is logically possible that p $=_{Df}$ p does not entail ¬p

or

(3) It is logically possible that p $=_{Df}$ there exists q such that p does not entail ¬q.

Because the eliminativist used the concept of entailment in definition of possibility he would then (in order not to be circular) have to come up with a rather syntactic explanation of entailment and adjust the rest of his logic accordingly. This approach to modality is quite rare[83] but might be interesting because it avoids completely any ontological and philosophical problems that the other approaches are vulnerable to. One possible worry is whether the possibility defined in terms of (syntactically understood) entailment would be strong enough to cover our notion of broad logical possibility, which we re interested in. Our main objection against both fictionalism and eliminativism is that they do not take the possible worlds semantics, i.e. the depraved semantics for modality, seriously enough. They might be suitable from formal point of view and may even have great explanatory power. But the serious treatment of depraved semantics is a criterion for an adequate semantics for modal logic they are not acceptable. And so fictionalism and eliminativism have to be rejected.

Modal realists and reductionists take on the other hand the depraved semantics seriously and construct systems where possible worlds are entities of some kind. Although there is some discussion about what does it mean to be a modal realist[84] we can clearly distinguish two approaches. Firstly, there is D. Lewis with his extreme modal realism and the claim that possible worlds are spatiotemporally and causally isolated realms, which in every respect exactly resemble our universe. Then there are (moderate) modal realists like Adams, Plantinga or Stalnaker. Modal realists acknowledge existence of possible worlds – the ways things might have been – but they treat them as abstract entities (world-stories, maximal consistent sets of propositions etc.) Modal

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[81] We roughly follow the Introduction of M. Tooley in (1999b) p. xviii – xxi.
[82] Tooley (1999b), p. xix
[83] In fact the only discussion of it I know about is in Lycan (1999). Lycan calls it the paraphrastic program.

realists in general reject the reductionism thesis that modal facts supervene on non-modal facts and, as Tooley puts it, they "postulate irreducible, modals states of affairs, and this, in turn, requires postulation of irreducible modal properties."[85] Modal reductionists then try to replace possible worlds by some (constructions out of) non-modal entities. The main attempts include construing possible worlds as sets of linguistic entities (R. Carnap), complex structured universals (P. Forrest), sets of space-time points or ordered quadruples of real numbers (W. V. O. Quine, M. Cresswell), rearrangements or combinations of some previously postulated metaphysical atoms (D. M. Armstrong, P. Tichý).

From our point of view the most "serious" treatment enjoy the possible worlds from D. Lewis. So let us start with his system and see whether it represents a sound and reasonable approach to modality. Lewis' theory is aptly summed up into few points by W. Lycan. According to him Lewis can be said to hold following theses:

(i) "There are non-actual possibles and possible worlds, and 'there are' here needs no scare quotes; non-actual possibles and worlds exist, in exactly the same sense as that in which our world and its denizens exist.

(ii) Non-actual objects and worlds are of just the same respective kinds as are actual objects and the actual world. Non-actual tables are physical objects with physical uses; non-actual humans are made of flesh and blood, just as you and I are.

(iii) Non-actual objects and worlds are not reducible to items of less controversial sorts; worlds distinct from ours are not sets of sentences, or mental constructs of any sort, but blooming, buzzing worlds.

(iv) Quantifiers range over not all the actual individuals that there are but all the non-actual ones that there are as well, unless their ranges are explicitly or tacitly restricted in contexts.

(v) All individuals, actual or mere possible, are worldbound; there is no genuine identity across worlds. You and I are not world-lines, but merely have counterparts in other worlds who resemble us for certain purposes but are distinct individuals in their own right.

(vi) Expressions which distinguish actual individuals from among all the possible individuals, such as 'real' and 'actual', are really relational expressions holding between individuals and worlds; an individual i is actual only 'at' or with respect to some world w. When we, in this or (our) world, call some object 'real' or 'actual', these terms are abbreviations for the

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[84] See for example Plantinga (1987) who claims approximately that even D. Lewis, the proponent of spatiotemporal possible world, is in fact modal reductionist because his truthmakers for modal sentences include only non-modal facts. But if Lewis is not a modal realist, then who is?

[85] Tooley (1999b), p. ix.

indexical 'real (actual) at our world'; every possible individual is real 'at' the worlds it inhabits."[86]

So Lewis postulates a (maybe incredibly) huge amount of physical but causally and spatiotemporally disjoint universes like the one we live in with inhabitants like us. These worlds can nevertheless be completely unlike the our including the type of its space-time (there can be worlds with endless time loop or such that the time slowly speeds up as well as spaces with different dimensions), its physical composition and laws (worlds composed of anti-matter), number and types of inhabitants (worlds with ghostly beings and flying horses, worlds where I am missing or have different properties) and so on. The expression 'actual' designates the world of utterance. So when I say

(4) It is possible that my actual mother is rich.

it means that the (one and only one) individual that is my mother in the world I inhabit is possibly rich in some other world which in turn means that in that world there is a individual (maybe completely unlike my mother), so called counterpart, that is rich at that world.

It is no dispute that Lewis treats possible worlds with utmost seriousness. But is this treatment adequate? It is difficult to believe that it should be. It has certainly many theoretical advantages, e.g. that it delivers probably the only non-circular account of modality or provides complete reduction of modal facts to non-modal facts. But the latter can be achieved in many different ways and it is not clear after all why the former should count as a devastating deficiency.[87] Let us therefore state some traditional objections to Lewis' theory that have been formulated over last thirty years. Because there are quite a few we will mention only those that directly involve the notion of possible world and omit (most of) those that criticize another part of Lewis' solution – the counterpart theory.

First of all, it seems that whole of Lewis' intuition for treating possible worlds as he does is ill-founded. When Lewis is persuading us that there are possible worlds he says: "I believe there are possible worlds other than the one we happen to inhabit. If an argument is wanted, it is this: It is uncontroversially true that things might have been otherwise than they are. I believe, and so do you, that things could have been different in countless ways. But what does this mean? Ordinary language permits the paraphrase: there are many ways things could have been besides the way that they actually are. On the face of it, this sentence is an existential quantification. It says that there

[86] Lycan (1999), p. 287-8. Similar presentation can be found in Stalnaker (1976), p.67-8.

[87] According to D. Lewis the acceptance of primitive modality by a theory of modality is a serious defect and prima facie objection to the theory in question. M. Tooley in (1999b, p. ix.), however, argues that it is normal in theories that certain properties are taken as primitive, i.e. that they are treated as irreducible to other properties or relations. In physics such irreducible property is be 'to have unit negative charge' and no philosophers wonder about it. So according to Tooley, to take necessity as an irreducible property of e.g. propositions should not be problematic either. One can also raise questions about closeness of formal and informal semantics as we did in section 4.1.

exist many entities of a certain description, to wit, 'ways things could have been'. I believe things could have been different in countless ways. I believe permissible paraphrases of what I believe; taking the paraphrase at its face value, I therefore believe in the existence of entities which might be called 'ways things could have been'. I prefer to call them 'possible worlds'."[88] Lewis' argument is very persuasive. It shows that to introduce possible worlds is nothing unnatural. After all, we all believe in them and have believed them long before Lewis. But for the argument to be successful "the shift from 'the way things might have been' to 'possible worlds' must be an innocent terminological substitution […]."[89] Stalnaker[90] argues that it is not. He remarks that the concept of possible worlds as 'the ways things might have been' is incompatible with Lewis' claim that possible worlds are like the world I inhabit (the causally and spatiotemporally maximal totality of which I am part) or as Stalnaker says like "I and all my surroundings". Stalnaker argues further that if possible worlds are ways things might have been, then the actual world should be also the way things are rather than I and all my surroundings. But the way things are is a property or a state of the world and not the world itself. The way things are can represent the world but cannot be identical to it. Moreover, if we admit that properties can be uninstantiated, it can be the case that the way things are could exist even if there is no world that corresponds to that particular way. Lewis' argument is therefore all right; it only does not support his treatment of possible worlds. Lewis' thesis listed above as (ii) rests on the mere equivocation between 'the actual world', in the sense of 'I and all my surroundings' and the sense of 'the way things are'. Possible worlds understood as ways things might have been should be some kind of abstract entities rather than concrete objects. Stalnaker who claims that the actual world is actual from the absolute standpoint and so is the only one that really exists rejects (ii) and goes for moderate realism. He says: "In fact, I want to argue, one must exclude those analogues of our universe from one's ontology. The thesis that the actual world alone is real is superficially analogous to solipsism – the thesis that I alone am real, but solipsism has content, and can be coherently denied, because it says something substantive about what alone is real. In effect, solipsism says that the actual world is a person, or a mind. But the thesis that the actual world alone is real has contents only if 'the actual world' means something other than the totality of everything there is, and I do not believe that it does. The thesis that there is no room in reality for other things that the actual world is not, like solipsism, based on a restrictive theory of what there is room for in reality, but rather on the metaphysically neutral belief that 'the actual world' is just another name for reality."[91] As a result, we can have serious doubts that Lewis'

[88] Lewis (1973), p. 84. Quoted according to Stalnaker (1976), p. 66.
[89] Stalnaker (1976), p. 67.
[90] Ibid., p. 68-9.
[91] Ibid., p. 69-70.

motivation is persuasive. It even seems to work against him in that it supports the idea that possible worlds are in fact abstract and not concrete as he supposes.

There are also more serious objections. First and most natural objection is that the totality of what exists is according to Lewis incomparably greater than we normally believe. As a result Lewis might seem to violate the principle of parsimony. But to be frank, Lewis is only postulating entities of the sort we already know. His possible worlds are of the same kind as the actual one. So this objection is not so strong. Secondly, there are certain problems with definition of concrete possible worlds and their distinctness. Famous objection is the one raised by Tom Richards: “No criterion is provided for recognizing [our] totality [of existents] as distinct from other totalities. The totalities are disjoint, but what divides them? Defining ‘x actually exists’ as ‘x belongs to the same totality as I do’ presupposes an entirely unexplained principle of identity for these totalities. Their contents, for Lewis, are all equally real [sic] so ‘our’ world cannot be distinguished by appealing to a difference between existents and possbilia”[92] The problem can be formulated as follows. You and I are in possible world w_a, Sherlock Holmes in w_b and Mackbeth in w_c. In virtue of what is Sherlock Holmes or Mackbeth not in our world? Because all worlds are concrete and independent on human imagination or stipulation, Lewis should provide some objective criterion for grouping of individuals into different worlds which he does not. Moreover as Richards and Lycan[93] show it is not so easy to do so. Maybe, and that would be Lewis’ preferred answer, worlds are divided by spatiotemporal isolation (and the metaphysical “glue” of the world is its spatiotemporal interconectedness), but what about nonphysical worlds, worlds that alone contain spatiotemporal or causal dislocations. Thirdly, ordinarily our intuitions about meaning of modal statements do not match those of Lewis. When we say and believe

(5) There might be talking donkeys

we have no reason for believing that there are some spatiotemporally disjoint worlds in which there are donkeys of flesh and blood that indeed talk. Similarly we do not consider following to be contradictory statement:

(6) There might have been talking donkeys even though there aren’t any, neither in our spatiotemporal universe nor in any other universe spatiotemporally disjoint from ours.

Mentioned intuitions are relatively robust and so when people are confronted with Lewis’ theory they side with those intuitions rather than with Lewis. Last objection from this group concerns the fact that all worlds exist in the same way. In our world, Einstein is for example the one who invented relativity. What distinguishes him from all other individuals is that he did so as a first one. But it is also possible that I were the first one who came up with that theory. As a result there is a

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[92] Lycan (1999), p. 25. Original argument and its elaboration to be found in Richards (1975), p. 108-9.
[93] Richards (1975), p. 109, Lycan (1999), p. 297.

concrete world with a concrete individual (my counterpart) who is the inventor of relativity. Now what distinguishes Einstein from that counterpart of mine? Normally we would say that Einstein is the one who in fact (actually, really) invented the theory and that the other individual did not. But on Lewis' account this difference does not count. For both Einstein and the other individual exist in the same way and invented relativity. But such a result is not what we want. We want Einstein to be the only distinguished (real or actual) discoverer of relativity.

Further objection is an epistemological one. It roughly states that if we are spatiotemporally and causally isolated from other worlds, how can we (i) form beliefs about their existence and more generally (ii) how can we know anything at all about them. Basic motivation is the following. We know that certain modal statements are true. (We are often able to verify that certain propositions or corresponding states of affairs are possible by producing them by our actions) This leads us together with correct analysis of modal statements to the belief that there are possible worlds. But if these concrete worlds are completely isolated, how can we ever have a faculty that enables us to form beliefs about their existence? Further how can we ever have any independent evidence for their existence? Lewis does not give us any informative answer. Moreover, it seems that there is no answer at all. The only possible answer is that worlds must be of the above-mentioned type because it is required by Lewis' account. This answer is not that bad as long as Lewis' metaphysical conception of possible worlds would be directly needed to form a functional semantics. But Lewis goes in his metaphysics far beyond this need and there might be alternative conceptions that provide equivalent basis for semantics. And so an approach with more moderate metaphysics should be given a preference.

Last but not least, there are also some technical objections to Lewis' account which we want to mention in brief. One objection concerns the reading of quantifiers. Normally, logic is equipped with one quantifier that ranges over existing objects. With this quantifier we can formalize our existence claims. So for example 'There are donkeys' is formalized as '$(\exists x)Dx$' and 'There are no talking donkeys' as '$\neg(\exists x)(Dx \& Tx)$'. But possibilist like Lewis (or Meinong) claim that "there are" also things, which do not actually exist. Expressed by means of standard logic, this is a plain contradiction in the form of

(7) $(\exists x)\neg(\exists y)(x=y)$.

So possibilist must somehow distinguish the two quantifiers so that one ranges over the totality of things that there are and the other is the standard one. Let's call the prior "Meinongian" and the latter "actualist". Now (7) can be stated in the form of

(8) $(\exists_M x)\neg(\exists_A y)(x=y)$,

which is no more contradictory. We can even with the help of one extra primitive – the predicate of actuality - define the actualist quantifier as

(9) $(\exists_A x) \equiv (\exists_M x)\text{Actual}(x)$.

(Note that we cannot define the Meinongian quantifier in terms of the actualist one, because its range is much greater than the set of actually existing objects.) This of course requires some extra semantics for the new quantifier $\exists_M$, which is by no means uncontroversial. Some people even claim that Meinongian quantifier (as objectual) is unintelligible, that it is "literally gibberish or mere noise"[94]. (There are also non-objectual readings of '$\exists_M$' but Lewis' worlds are genuine objects and so the quantifier must range directly over them.) Another technical objection that we will only mention is the one of P. Forrest and D. Armstrong in (1984). It says that given Lewis' account there cannot be anything like set or aggregate of all possible worlds.

To conclude we have to say that although Lewis' extreme modal realism account of possible worlds certainly takes possible worlds at face value it seems to exaggerate concerning their mode of existence. Extreme modal realism is open to many objections and violates too many intuitions. It is of course price that one must pay for its theoretical advantages, but it seems that it is too high. Let us conclude the discussion of Lewis' account by short quotation of Lycan: "The […] drawbacks […] are neither individually nor jointly decisive against Lewis' position. I do not believe that it is possible to refute Lewis' position, unless the enormous cardinalities involved in system of possible worlds should trigger some ingenious diagonal argument that is beyond my mathematical expertise to devise. I do think that the drawbacks are serious enough to provide strong motivation for seeking some alternative modal metaphysics."[95] We share this view with Lycan and so we better look for some other system. Prior to doing so we will, however, establish some principles that our system should match. We start with the relation of the actual world and its non-actual alternatives.

4.2.2. Actualism

In previous chapter we have rejected Lewis' position, which treats non-actual possible worlds as existing irreducible concrete entities and postulates non-actual concrete individuals. Such approach is sometimes called possibilism. In this chapter we would like to defend the opposite view that can be aptly described by thesis

(A) Everything that exists is actual.

We will call such position actualism. Actualism itself is very intuitive approach to reality and existence. It certainly surpasses possibilism in matching commonsense intuitions. Actualism can be aptly illuminated by following imagination. Imagine Leibnizian God who has in his perfection

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[94] Lycan (1999), p. 18.

[95] Lycan (1999), p. 25.

understanding of infinity of possible worlds. He, however, actualizes just one. As a result there is just one actual world – synonym for everything there is, synonym for reality. Actualism concerning possible worlds adds that everything non-actual has to be constructed out of parts of the actual world. So much for brief introduction. But before we move on to elaborate the actualist position, we have to make several distinctions.

Problems of actuality can be discussed on several levels. First of all, we have to distinguish the semantical or logical question from the metaphysical one. The semantical question is: "How shall we categorize, within the framework of some semantic theory, and characterize the specific content of, words like 'actual' and its cognates?"[96] So the semantical question concerns meanings of some expressions of given language. A. Hazen in (1979) considers two approaches that have some adherents: realization theory (actuality is the property of worlds possessed by each possible world only in itself and in no other), indexical theory (the (English speaking) inhabitants of each possible world call that world and no other actual similarly to the way every (English speaking) person calls himself and no one else 'I'). He also lists one theoretical option - the name theory - according to which 'the actual world' is a proper name of the actual world. This semantical question has to be distinguished from the metaphysical question which itself can have two aspects. First we can ask what is for a possible world to be the actual world. Secondly there is a question about what is it for a thing to exist in a given possible world. The prior we shall call in accordance with R. M. Adams the problem of actuality, the latter the problem of existence. In what follows we will ignore the problem of existence and will focus on the problem of actuality and on the semantics of 'actual'.

The semantic part is quite clear. There is a broad consensus among philosophers that the semantical part is independent on the problem of actuality[97] and as a result can be (and also is) combined with different metaphysical attitudes.[98] We will all agree that the function of 'actual' is to isolate some part of the expression (formula or sentence) from its surroundings and "remove it" from the scope of or "protect it" from modal operators. As a result we should get '$\Diamond(\text{actually}(\varphi) \,\&\, \psi) \equiv (\varphi \,\&\, \Diamond\psi)$' as valid. Similarly to match our intuitions the inner quantifier in 'There could have been other things than there actually are' should be protected from the influence of modal operator and should range constantly over the objects of the actual world rather than (as in the case where 'actually' would be missing) over the same individuals as the outer quantifier. Out of Hazen's list realization theory, which he attributes to Plantinga, does not fulfill this criterion. According to it every world is actual in itself and so 'actually(φ)' is true at w iff φ is true at w. But then 'actually'

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[96] Hazen (1979), p. 1.
[97] See for example Hazen (1979) or Stalnaker (1976).
[98] For example the indexical analysis of 'actual' is defended by Lewis in (1986) who also defends extreme modal realism and concrete possible worlds, by Stalnaker in (1976) where he proposes to treat possible worlds as abstract

does not isolate any part of the expression and '◊(actually(φ) & ψ)' is equivalent to '◊(φ & ψ)'. The name theory as well as the indexical thoery fulfill the above criterion. How shall we then decide between them. I would side with Hazen and say that even though there can be a possible world whose inhabitants use 'actually' to refer to our own world it is rather implausible. Expressions like actual are unlike proper names of individuals parts of the language. So if the inhabitants of some possible world use 'actual' with the same meaning as we do they use it to refer to their own world. As a result the indexical theory is the most accurate.

There can be one more advantage of the indexical analysis of actuality. Sometimes it has been argued (especially by philosophoers whose systems do not operate with distinguished actual world) that we are not in the position to know which possible world is represents the actual world.[99] Our knowledge is undoubtedly finite and we can know only limited amount of facts about the actual world. Further, possible worlds are perfectly determined, i.e. for every proposition p, either p or ¬p is true at w. As a result there might be many possible worlds that are compatible with what we know about the actual world. To know which one is (or corresponds to) the actual world would mean to know every fact (possibly including those future ones) about it which is equivalent to omniscience. But people are certainly not omniscient. To treat 'the actual world' as for example proper name of certain world (and to know what we are saying i.e. to know the referent of this name) would imply omniscience. Indexical analysis saves us from this problem because when we say 'the actual world' we mean the one we are part of and it is left up to the facts which one it really is. Note that the analysis of 'actual' is on the metaphysical claim that every possible worlds is equally real.

So much for the semantical part. But what about the problem of actuality? What does make the actual world actual from the ontological point of view? We might be tempted to go for the indexical solution as well and say that actuality is a world-relative attribute which our world has in relation to itself similarly to other possible worlds. As a result actuality does not, we would say, distinguish our world from the absolute standpoint from any other possible world. This is typical possibilist approach. Possibilists usually start with the whole system of possible worlds and see the actual world primarily as one of them. Moreover they consider the absolute metaphysical standpoint to be a neutral one. They think it is distinct from the standpoint of any possible world. I would like to join Adams (1974), Stalnaker (1976) and others and claim that this approach is inaccurate. The reasoning should be done the other way round. First of all, there is the actual world, i.e. our world. Secondly, its standpoint is the absolute one and it alone is actual. And thirdly, the whole system of

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irreducible entities – the 'ways things might have been', by Tichý in (1996b) who otherwise goes for combinatorial account of modality or finally by Hazen in (1979) whose metaphysical attitude is unknown to me.

[99] Such argumentation can be found for example in Tichý in (1996b), p. 53 – 7.

so-called non-actual possible worlds with non-actual individuals is (or shall be) part of the actual world.

But why shall we follow the actualist rather than the possibilist with his indexical actuality? First of all, there is massive actualist intuition that actuality is a distinguished metaphysical status. We normally believe that things that exist actually are more real than those that do not. So the possible house (I plan to build) or money (I might get at the end of the week) are less than real house that is already built or money I got yesterday. This concerns individuals as well. As Adams puts it: "We do not think that the difference in respect of actuality between Henry Kissinger and Wizard of Oz is just a difference in their relation to us."[100] But that is exactly what indexical theory indicates. According to it both Kissinger and Wizard of Oz are concrete objects and exist exactly in the same way. Only Kissinger belongs to the same world as you and I whereas Wizard of Oz is spatiotemporally and causally isolated from us. (Wizard of Oz can naturally claim in his world that he and his worldmates are perfectly real. And he can be as right as we are when we do so. But his standpoint is fictional so it does not after all make any difference with regards to reality.) Another Adams' argument is an ethical one.[101] We normally think that it is not bad that there are evils in non-actual possible worlds nor good that there are joys. We might be, of course, moved by suffering of a fictional character but we do not take it at face value. But it would be bad (and good respectively) if these evils (and joys) would be actual. So for example our condemnation of misdeeds may be a good illustration of the belief that the actual (and realization of things in the actual world) has a special status. Finally, actualism represents a reasonable and modest ontology concerning concrete particulars. It does not forces us to acknowledge anything concrete (e.g. merely possible concrete objects) that would exist outside of our physical world. As such it represents a reasonable base for intuitively acceptable modal system.

Let us now sum up Adams' actualism. He claims that actuality is absolute property that distinguishes our world from all other possible worlds. He further defines actualism with respect to possible worlds as "the view that if there are any true statements in which there are said to be non-actual possible worlds, they must be reducible to statements in which the only things are said to be are things which there are in the actual world which are not identical with non-actual possibles."[102] This definition includes two important principles. Firstly, the unactualized possible worlds have to be construed out of "furniture" of the actual world. That means that the furniture has to be rich enough to allow for successful construction of such system of possible worlds that would provide

[100] Adams (1974), p. 215.
[101] Ibid., p. 216.
[102] Ibid., p 224.

adequate analysis and truth-values to all modal statements. Secondly, the notion of non-actual possible world must not be taken as primitive.

We shall not investigate into details of Adams' 'true-story theory of actuality'. Instead we would like to draw some general principles from the previous paragraphs. Firstly, we agree with Adams that actuality is an absolute property that distinguishes our world from any other. However, this does not necessarily prevent us from analyzing 'actual' as an indexical. Moreover we think it is the correct analysis. Concerning Adams' actualism with regards to possible worlds we find it sound to insist that non-actual possible worlds shall be part of the actual world but we do not think that they necessarily have to be reducible to something else. Maybe the solution with reducible possible worlds shall be preferred but we certainly do not violate any actualist principles if we treat possible worlds as irreducible abstract entities e.g. Stalnaker's 'ways things might have been'. (The informative value of such thesis is another question.) If we go for composite possible worlds we can construct possible worlds out of states of affairs, propositions or as recombinations of some metaphysical atoms. Our first principle for construction of acceptable modal system will then be that any such system should contain only abstract non-actual possible worlds and should contain one distinguished actual world.[103] And if possible we should also avoid assigning of properties to individuals in worlds in which they do not exist.[104]

4.2.3. Haecceitism And Trans-world Identity

Another important problem for a modal system are individuals and their identity. Although the metaphysical conception of individuals is of great importance we shall rely on some intuitive treatment and omit them here. We shall focus above all on the problem of identity and so-called trans-world identity. In the following section we would like to defend the view that the (trans-world) identity of actually existing individuals is primitive and that all the problems with it are brought about by a misconception of identity and the function of names (so-called telescope view). However, non-actual individuals that differ from all actual ones have to be added as abstract objects.

[103] In the end small remark on one drawback of actualism about possible worlds. If we understand possible worlds as constructions out of propositions and decide to analyze actuality in terms of truth (like Adams does), i.e. we analyze 'In the actual world p' as 'The proposition that p is true', we might be forced to distinguish between the notion of truth and the notion of truth-at-world and eventually take the notion of truth as prior to the notion of actuality. We see this as a minor problem. Because we hold that the correct analysis of actuality is the indexical one we will not pursue this problem further. For more see Adams (1974), p. 225 – 30.

[104] This principle is called serious actualism. We can express it formally by $\Box(\varphi(x) \rightarrow (\exists y)(y=x))$ where φ is atomic statement.

Let us first explain where does the problem lie. In our formal sketch of semantics for modal systems in section 3.2, we have admitted possible worlds with (partially) overlapping individual domains. As a result one and the same individual can appear in different possible worlds. Let us then suppose that some individuals from world w_1 are also present with slightly altered descriptions in w_2. We will follow the famous example of Chisholm and take Adam and Noah as two of these reoccurring individuals.[105] Say now that in w_1 Adam lives for 930 years and Noah for 950, but in w_2 Adam has the property of living 931 years and Noah lives for 949. We are immediately confronted with the first problem. If indiscernibility of identicals is true how can Adam of w_1 be identical to Adam of w_2? They after all differ in age? Following similarities with the notion of trans-temporal identity we can deliver a straightforward answer. Concerning identity through time we can also doubt how can the young Adam who met Eve for the first time be identical to Adam at the age of 929? There also is a difference in properties. For example young Adam is childless but the old man has many children. But this is all right. With the help of time indexes we can say that Adam has the property of being childless at some t_1 but has many children at some (later) t_2. And these two properties are certainly not incompatible. The situation with possible worlds is now analogous. To have property of living for 930 years at w_1 is compatible with living for 931 years at w_2. So far so good. Imagine, however, a different possible world w_3 where Adam is 932 and Noah 948 years old and another where Adam is 950 and Noah 930. Imagine also some sequence of worlds ending with w_n where Adam and Noah exchange one by one all of their properties including finally their names. So Adam of w_1 is now, so to say, Noah of w_n and vice versa. Here we are facing the second, more serious, group of problems. Is Adam of w_1 identical to Adam of w_n or Noah of w_n? How can we identify our Adam in w_n when we have no direct access to w_n? And even further, how can we tell the difference between w_1 and w_n when they are qualitatively identical and differ only in respect to individuals?

The traditional answer to these perplexing questions is essentialism. Essentialist would say that among properties things have, some of them are essential, i.e. such that a thing cannot loose them without ceasing to be the thing it is. More precisely P is essential property of x iff necessarily x has P in all possible worlds in which it exists. The notion of essential property can be now used to define so called individual essence. E is an individual essence of x iff E is essential to x and there is no possible world in which there exist an object distinct from x that has E. Classical addition to this definition is that such an essence is (at least partially) qualitative. That means that it includes some properties that are purely qualitative.[106] The reason is that there are many so-called trivial properties

[105] Chisholm (1967), p. 1- 4.

[106] By purely qualitative property I mean roughly a property that is so to say general (can be exemplified or possessed by more different individuals) and not relational (it does not include reference to other individuals). Examples of such

that are necessarily possessed by every individual, e.g. to be self-identical or to be colored if red. But if the individual essence consisted solely in these trivial essential properties, it would be of no help in identifying the individual. Equipped by above-mentioned notions we can now identify according to essential properties or preferably individual essences. Two objects in different possible worlds are identical iff they have the same essence or alternatively iff they share all the essential properties. This criterion gives us now direct answers to our previous questions or better said it blocks the production of such property-exchange examples like the one with Adam and Noah. According to essentialist,

This view can be described by the famous title – "the telescope view". We shall probably explain this. Let's take then the statement

(10) It is possible that George W. Bush is bald

and translate it into our possible-world talk. We get

(11) There is a possible world w such that G. W. Bush is bald at w.

But in order to determine the truth-value of (11) we have to first figure out which individual is identical to G. W. Bush at w. We would directly say, that it is G. W. Bush himself that is this individual at w. But that is not so easy for remember that the description of G. W. Bush might be altered in many ways and moreover we have no direct access to w. We are in a position of an observer that looks at this world through a telescope. He can observe the world, carefully examine every individual and its properties (now the importance of qualitative essential properties becomes apparent) but he cannot be directly acquainted with any of them. So the decision as to which individual is (or is identical to) G. W. Bush has to be made on purely qualitative grounds. And here essences and essential properties come to play. So in the end (10) is true iff

(12) there is a world w and individual i such that i has the same essence as G. W. Bush (or has the same essential properties) and i is bald.

Alternative solution to the trans-world identification problem would be to prohibit overlapping individual domains at all. According to this view every individual can occur in just one world. Then there can be, of course, no identity between individuals in different possible worlds. Trans-world identity is then usually replaced by some weaker intransitive relation, e.g. similarity or some more sophisticated counterpart relation. We will not elaborate on this but given counterpart theory (10) is true iff

(13) there is a world w and individual i such that i is the counterpart of G. W. Bush and i is bald.

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properties are 'to be blue', 'to be beautiful' but not 'to be higher than John' or 'to live 2 miles east of London). I will deliver precise definition of qualitative property later in this chapter.

There are two main proposals for counterpart relation which were defended in the literature. One candidate is the relation of similarity the other is the proposal that we can design counterpart relations more or less according to our theoretical needs.[107]

We find both above-mentioned solutions of the trans-world identity unsatisfactory. The counterpart theory violates our original understanding of possible worlds – ways things might have been. Of course there are ways things might have been that include only individuals that do not occur in our world. But there certainly are such ways, e.g. where I am famous musician, that include me as well. Moreover it is at least unnatural to say that all my modal properties have nothing to do with me and depend on (often contingent) properties and activities of some individuals in other worlds that are often completely unlike me but are identical to or are counterparts of me. The essentialist solution on the other hand invokes the notion of essential property. But it is not certain whether there are any non-trivial essential properties. And even if there are we seem not to be in position to know it. Which for example are the essential properties and what is the essence of the actual G. W. Bush? Is he essentially the President of the U.S? Certainly not because every individual can have this property for maximally 8 (consecutive) years which have to be preceded by at least 21 years possessing it. But what about being male or being a person? Here we would not be so sure. Similar problems occur for qualitative individual essences. These problems aside, we claim that the whole problem of trans-world identity is a pseudo-problem generated by the telescope view of possible worlds. We will do so by showing that trans-world identity is a primitive notion. To understand it we must only get rid of improper understanding of certain semantical notions and identity of individuals.

Let us start with the semantic notions. To properly understand identity of individuals it is important to understand the behavior of expressions that we use to refer to them. Our language is for this purpose equipped with special category of expression we call singular terms. These can be (at least syntactically) distinguished into proper names and definite descriptions. Names are expressions like ‘Ondrej Tomala’ or ‘G. W. Bush’ that are used to pick out an object without (explicitly) mentioning of any of its features. Definite descriptions on the other hand pick out an individual by its unique trait. (We could say that definite descriptions are expressions of the form ‘the so-and-so’ where the is in the singular e.g. ‘the king of France’. But there are also expressions like ‘my favorite joke’ that function like definite descriptions but do not respect this grammatical form.) It is important to understand that the difference between these two categories of objects –

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[107] Counterpart relation as similarity is defended by Lewis (1973) <!!!find some reference!!!>. The couterpart relation as we need it used to be defended by Kaplan in (1979). Both Kaplan and Lewis later modified their views. Lewis in (1986), Chapter 4.3 defines worldbound individuals as (mereological) stages of trans-world individuals and adds that two stages are counterparts iff they are stages of the same trans-world individual. Kaplan in (1975) converted to haecceitism, a doctrine we shall describe later.

names and descriptions – is a fundamental feature of language. While descriptions pick out objects by their properties, names do so directly. Moreover, names are rigid designators.[108] Once they are introduced into language and once their reference is fixed names refer to the very same individual as long as it exists (and in the actual world normally also after the individual ceases to exist). There are many arguments why this is the correct view on naming. Interested reader may see works of S. Kripke or K. Donellan and many others.[109] I would like to explicitly mention just one motivation presented by Føllesdal (1998). According to him world is usually conceived (at least for systematization purposes) as being composed of objects. Objects have at least three distinctive features: (i) they are bearers of large number of properties and relations some of which we know of and some of which we do not, (ii) they undergo changes over time but remain identical through them, (iii) we may have false beliefs about them but while we try to correct them these beliefs are about the objects in question and not about any object that satisfy our beliefs. We also believe that the world as we perceive it may consist of more objects than we belief, the number of these objects is subject to change in time and that we may be mistaken about the objects there are and whether we confronted with the same object in different situations. Considering this we should expect that language contains a type of expressions (among them names) that are designed to refer to objects and stick to them no matter what happens and no matter what we believe. These genuine singular terms are, so says Føllesdal, introduced into language (even as a replacement of some definite description) because (i) we are interested in further changes of the object that go beyond the descriptive content of the original description, (ii) we want to track the object through changes, (iii) we are aware that some beliefs about the object are to be corrected. But after the introduction the ties with the descriptions can be broken. So for example the name 'meter' was established to denote certain object (with many interesting properties) which was believed to be the 10 000 000th portion of the length of the Earth's quadrant. Later when it was discovered that this belief is false, scientists corrected their beliefs about object called 'meter' and not the denotation of the term.

Another question is of course how a theory of such rigid expressions should be built and especially in virtue of what is their reference fixed. The discussion thereof would certainly require separate study and cannot be covered here. The results important for our further inquiry is that among singular terms there are expressions (including proper names) that behave like rigid designators. Notice also that we have established the notion of rigid designator without any explicit use of modal notions. As a result we can say that rigid designator is not essentially a modal notion. Further important result is that if possible worlds are constructed as abstract objects out of what actually exists (including individuals), and if names rigidly refer to (even no longer existing)

[108] See footnote 69 for definition.
[109] Kripke (1996), Donellan (1974) or Føllesdal (1998)

individuals of the actual world, then they also refer rigidly to those individuals in alternative possible worlds. In anticipating next paragraph we can say that with proper names we can express what we may call "thisnesses" of objects. And we may do so "without understanding each thisness (the property of being this or that individual) in terms of some other property or properties better known to us, into which it can be analyzed or with which it is equivalent."[110] If we are inclined to believe that there are such properties as thisnesses and that names are rigid designators we may say that thisnesses are semantically primitive. But what about the metaphysical part? Are there any non-qualitative thisnesses?

To answer this question let us first define some notions. The belief that there are thisnesses can be understood as a part of broader doctrine that it is meaningful to "speak of a thing itself – without reference either explicit, implicit, vague, or precise to individuating concepts (other than being this thing), defining qualities, essential attributes or any other of the paraphernalia that enable us to distinguish one thing from another."[111] D. Kaplan calls such doctrine haecceitism.[112] When it comes to differentiate the views that can be hidden behind that title things are not that easy. First of all we shall distinguish haecceitism with respect to some type of entity. One does not necesariy have to be haecceitist with respect to everything. Our concern will be individuals. One typical view is that there are non-qualitative essences of individuals (properties of being identical with a certain particular individual) – so called haecceities or thisnesses. Proponents of this view may differ as to whether haecceities are ontologically dependent on individuals or whether they can exist independently. Another view that I think is compatible with our definition of haecceitism is the view that individuals are simply or barely numerically distinct. We believe that even bare individual has some trivial properties. But my property of being identical to myself is in a sense trivial one. I cannot exist without having it. Moreover, every individual has it necessarily or not at all. So even if I am in fact bare individual, it is property I cannot lack. We also prefer a view that instead of postulating bare identity of unknown kind gives some informative answer of individualization.

In the following paragraphs I would like to argue that haecceitism with respect to the individuals of the actual world is an attractive and plausible doctrine. Nevertheless it has to be

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[110] Adams (1979), p. 6.

[111] Kaplan (1975), p. 723.

[112] Note that there is some perplexity in terminology about haecceitism. Lewis (1986, p. 221) for example defines haecceitism as the doctrine that there are at least some possible worlds that differ in what they represent de re concerning some individual but do not differ qualitatively in any way. (Imagine for illustration two possible "mini-worlds" w_a = {president(Bush), philosopher(Kripke)} and w_b={president(Kripke), philosopher(Bush)}. Now w_a and w_b do not qualitatively differ for in both there is one individual that is a philosopher and one that is a president and that's it. They however differ de re, because it is Bush that is president in w_a and not in w_b). This notion of haecceitism is of course independent on the view of proper names or individuating of objects. The only thing I can say here that haecceitism is not primarily a modal notion. It is a doctrine of individuation of certain entities and as such can be applied on broader scope of problems than trans-world identity. As a result I would prefer Kaplan's view. But it is after all only terminological dispute. So we should not pay much attention to it.

abandoned it once we start to think about non-actual individuals. Let me first define one useful notion. We shall say with Adams that a property is a suchness iff “it could be expressed, in a language sufficiently rich, without the aid of such referential devices as proper names, proper adjectives and verbs (such as ‘Leibnizian’ or ‘pegasizes’), indexical expressions, and referential uses of definite descriptions”.[113] Thisness is as we have said a (non-qualitative) property of being this or that individual. A relational suchness is now any property that is not thisness and that includes some relation to other individuals (e.g. being greater than). With this gear we shall now introduce first argument in favor of haecceitism. It is the famous argument of M. Black[114] against bundle theory of individuals and thus against the doctrine of identity of indiscernibles. Let me paraphrase it here. We shall imagine a universe consisting of two large perfectly spherical solid iron globes (and nothing else) that always have been, are, and will be exactly similar in every qualitative respect (shape, size, color, etc.) They even share every relational suchness, e.g. being two diameters from another similar iron globe. Since this universe is logically possible we have two qualitatively indiscernible things and identity of indiscernibles has to be abandoned. Similar example can be devised with time. Imagine a world of one-way ethernal recurrence, i.e. world where every epoch if followed by another one that is qualitatively indistinguishable. Now take an individual from epoch 5 and 10. They are perfectly alike but are separated by temporal distance. Again because such picture is perfectly possible identity of indiscernibles has to be dropped.

Another way of supporting haecceitism (especially for those who find Black’s example too stange) is the argument from almost indiscernible twins[115]. Imagine picture similar to Black’s only with the difference that now one of the iron globes is slightly different in non-relational way. For example one globe has color c_1 and the other c_2. This should be perfectly acceptable even for those who objected to Black. Call this world w_1. Now move on to the new world w_2 that is an exact copy of w_1 with one exception – both globes now have color c1. The result is again perfectly possible universe (obtained from a possible world by altering one property of one globe) with two indiscernibles.

I believe that these arguments are strong enough motivation for several conclusion. First, the standard modal formulation of identity of indiscernibles

(14) $\Box((\forall x)(\forall y)(\forall F)(Fx \equiv Fy \rightarrow x=y))$

where F can stand for all suchnesses (including realtional) is not true. It might be the case that the non-modal version holds as a contingent principle for the actual world. But the philosophically

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[113] Adams(1979), p.3. On the same place Adams provides a more precise recursive definition of suchness. But the one presented should be sufficient for our purposes.

[114] Black (1952), p. 156 ff.

[115] Adams (1979), p. 17.On the same place Adams also has more complex version with minds, which we do not reproduce.

interesting modalized version does not. Secondly, individuals cannot be individuated on a purely qualitative basis. There has to be some further metaphysical component – a property that is responsible for individuation – a non-qualitative thisness. Thirdly, as a by-product, we get a very plausible notion of trans-world identity. The reasoning goes as follows. By refuting identity of indiscernibles we have shown that thisness are not expressible by means of suchnesses and as such are metaphysically primitive. To be for example G. W. Bush is the same property in every possible world. That means that if it is different from every suchness in one possible world it cannot be identical to one in another. It would be bizarre to claim that the thissness in question is distinct from all suchnesses when it is exemplified by the President of U.S. in the actual world but is identical to some suchness when it is exemplified by the looser in last presidential election in some alternative world. The result is that thisness is non-qualitative either in all possible worlds in which it occurs or in none. Now thisness is present in all worlds in which the individual is present and as such it guarantees its identity and distinctness. It also guarantees that there are distinct possibilities for distinct. Let us return once more to our two-globe universe from above. We have two indiscernible globes in otherwise empty space. For simplicity, let us call one g_1 and the other g_2.[116] There can be two qualitatively indiscernible worlds w_1 and w_2 such that in w_1 g_1 ceases to exist at some t and g_2 last forever and in w_2 things are the other way round. Let us now suppose that there is an inhabitant on g_1 (with indiscernible twin on g_2). Now although there are no qualitative differences between w_1 and w_2, from the point of view of the inhabitant of g_1 w_1 and w_2 make a difference. It is after all him who is annihilated at w_1 but not at w_2. But there is no way of how to qualitatively construct the trans-world identity and non-identity for g_1 and g_2 and not conflate w_1 and w_2. The conclusion is that qualitative similarities cannot explain different trans-world identities for indiscernible individuals nor the identity itself.

So from the metaphysically primitive thisness we have deduced that the notion of trans-world identity must be primitive as well. Each individual has its thisness and keeps having it in every possible world. This thisness then serves as a "handle" for this individual. Our two results about thisness – the semantical and metaphysical primitivity – now entitle us with regards to possible worlds to construct them with explicit reference to individuals they contain. We do not have to limit ourselves to qualitative description and consider the identity of individuals to be what Kaplan calls "an artifact of the model"[117]. No, the identity of individuals is a metaphysical feature and we have full right to say that two mini-worlds {president(Bush)} and {president(Gore)} differ in that one contains Bush and the other one does not. Similarly the statement

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[116] We loosely follow Adams (1979), p. 18. Someone could object that by giving names we spoil the qualitative indiscernibility. Not necessarily. Convenient way is to think about them as of informal counterparts of variables bound in existential formulas that would be used to describe the example in a formal way. (Ibid., footnote 24)

(15) It is possible that G. W. Bush can't play golf
is true iff there is possible world w where Bush himself (not an individual with same core qualities, not his counterpart) is a poor golf player.

So far for the actual individuals. But our understanding of possible worlds allows that there are worlds where there are extra individuals or where some from the actual individuals are missing. The latter is no problem. All actual individuals have their identities fixed in the actual world and so a contracted world is not an issue. The augmented world however presents some difficulties. We leave aside the situations in which some actual individual has some new unknown property (produced by some logical operations from those we know) or when a new aggregate is built from existing individuals. We mean a situation when there is an individual distinct from all actual individuals. We have, however, committed ourselves to actualism and so we cannot deal with merely possible (or future) individuals on a par with the actual ones. According to our claim merely possible individuals have to be similarly to merely possible worlds "constructed" out of what there actually is. All right, but does this ban us from treating them on a par with actual individuals? We can be tempted to add for example Pegasus by analogy to the actual horses, endow him with property of having wings and the like and secure his individuation by saying that it is as individual simply numerically distinct from the other individuals. A possibilist could do so without any problem, an Actualist cannot. Especially when we accepted primitive non-qualitative thisnesses.

At this point we have two or three options. First option is to claim that merely possible individuals cannot have any thisness. To have thisness is a distinguished feature of the actual. Moreover thisness is related to the respective individual – my thisness for example to me – and cannot exist without being mine. This is all right for the actual individuals but not for merely possible ones. We find this option close to common sense and therefore also quite attractive. The drawback is that we are forced to postulate contingent universals. The second option is to say that thisnesses are prior to individuals and that they can exist independently on them. In what follows we shall call such thisnesses haecceities. Now it is no longer individual that is the basis for identity but the haecceity itself. We would get rid of contingent properties but we would have to admit that there are many uninstantiated haecceities. In the rest of this section we would like to argue that the first approach is more natural and that there are decisive objections against the other.

First of all let us put down some reasons why we find haecceities problematic. First of all it is difficult to say what haecceities are. Suppose for example there is a haecceity H of G. W. Bush. Suppose now that G. W. Bush never existed. How can than H be his haecceity? We do not deny that it is haecceity of something. But why exactly his? Maybe it is possible to go over this somehow.

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[117] Kaplan ….

Suppose then that there are really necessary properties that an individual has or is related to essentially but not vice versa. But if it is so what in reality does guarantee that every haecceity is used only once or metaphorically what does prevent God from using the same haecceity twice? A nice problem with haecceities arises when we consider fictional characters. Adams mentiones Sherlock Holmes. He wants us to imagine a case when long before Doyle's stories some retired schoolteacher in Liverpool writes story about an amateur detective called Sherlock Holmes with features so similar to Doyle's hero that if Doyle had published the story it would have been certainly accepted as story about Sherlock Holmes. Unfortunately, the story went unpublished and after the author's death the manuscript was burnt without being read by anyone but the author. Now, was the schoolteacher's story about Sherlock Holmes or not. A fried of haecceities must insist that it is an open question. It depends on whether Doyle and the schoolteacher happen to attach the same haecceity to their characters. Our intuition might be different. We might be (and I would say rightly) inclined to think that the story was not about Sherlock Holmes because one of the features that distinguish him from other heroes is that he bears an intimate relation to sir A. C. Doyle and his stories. Another problem is the one of distinct possibilities. Imagine again our universe with two indiscernible globes, this time as a merely possible world. Suppose further that any of the pair of possible globes is different from all actual individuals and that none of them bears any relation to any of actual individuals. Members of such a pair cannot have any qualitative essence nor any essence that would individuate them with respect to the actual world. Let us now ask, which one of the members do we wish to annihilate. But such a question seems to be problematic. The only description of this world is the existential statement

(16) $(\exists x)(\exists y)(Gx \ \& \ Gy \text{ and } x \neq y)$

where G stands for a complex property of being a globe of certain size, shape, etc. But (16) provides no grounds for identification and thus also no grounds for deciding between our globes. Probably not even a superhuman mind could make a choice. An haecceitist must claim that there is a way how to make the decision and that the possibility of one globe ceasing to exist rather than the other is a distinct one for each of the globes. Moreover he must claim that the person making the choice is somehow acquainted with the haecceities.

All together one might suspect that haecceities are postulated as entities to cover certain theoretical gap in theory of individuation of individuals. But if haecceity is only some "we do not know what" to fill some metaphysical role, then facing the above-mentioned problems it might be wise not to commit oneself to their existence and to make do with thisnesses whose metaphysical status is a bit clearer. Rejecting of haecceities also allow us to say that the possibilities for merely possible individuals are qualitatively indiscernible and that we cannot distinguish between them by using referential expressions such as names. (If we however decide to model merely possible individuals on abstract objects, we can use names for describing possible worlds containing them. These alleged names will be nevertheless nothing more than abbreviations for a bundle of definite descriptions because abstract objects are individuated qualitatively.)

To conclude this section let us summarize its results. After having rejected the telescope view we have argued for haecceitism, more precisely for the view that there are primitive thisnesses (properties not reducible to any more basic qualitative properties) that are ontologically dependent on individuals which they are thisnesses of. We have claimed that this view is not directly associated with the notion of trans-world identity, but once accepted it provides prima facie reason for the view that trans-world identity is also primitive and that the trans-world identification is not an issue after all. Concerning merely possible objects we have however adapted a different view, namely that they cannot be distinguished with the help of thisnesses and that the only way how to individuate them is the qualitative one. Although it is a theoretical complication it is a natural view. It secures that all possibilities concerning individuals from the actual worlds remain discernible. There is no conflation there. Concerning the merely possible objects and their possibilities we "conflate" possible worlds that are isomorphic with regards to qualitative descriptions of their parts with merely possible objects. I put conflate into quotation marks because it is after all no conflation. What we normally mean by saying that there could have been Pegasus (meant as 'the only flying horse') is after all that if the world with Pegasus had been actual, there would have been an individual that would exemplify all the properties we know from Greek mythology. And as such it would have had a thisness. But as a mere role description it cannot have any. Are we then ready to say that there could be two winged horses, Pegasus and Megasus both missing in the actual world who would be qualitatively indiscernible? (Do not forget that the names are mere abbreviations for the very same description 'being the winged horse that …'.) I do not think we are ready to allow for such situation. We would either say that there is after all some qualitative difference between them or that both names denote the very same object. (Merely possible objects are also often incomplete so that they would require some completion before they could become actual. So for example Pegasus could be any individual that has some required features but also many other that are not important for his being one.) Even less can I see how would we like to distinguish for which of the qualitatively indiscernible horses is it at some time t possible to fly eastbound and for which westbound. So there cannot be talk of any conflation because there are no possible worlds to conflate.

4.2.4. Relative Possibility And Other Issues

Last notion we would like to explore is the one of relative possibility. In our formal sketch of modal logic we have seen that the formal models make use of the notion of accessibility. Accessibility was defined as a relation on the set of possible worlds. Intuitively accessibility restricts the number of possible worlds that count as a valid alternative to some world. The notion of accessibility is of a great importance in modal logic because it allows us to interpret '□' in many different ways. We may for example decide to interpret '□' as 'it is known that'. The advantage is that we can use the same formalism as for any other modalities and even the same type of models. We only have to choose some suitable accessibility relation (in this case one that is reflexive, but not transitive nor symmetric) and accept related axioms.[118] Similarly we can obtain interpretation of '□' as 'it is provable'. But we can use the notion to introduce different kinds of alethic modality as well. So for example the notion of nomological modality can be obtained by introducing some (probably very complex) relation of accessibility.

But what about the broadly logical modality? What kind of accessibility relation do we need there? It is plain that if something is necessary it also has to be actually the case. So reflexivity is a natural requirement. But what about the other features? Consider transitivity first. What does the related axiom '$\Box p \rightarrow \Box\Box p$' in fact say? It expresses the notion that if a proposition is in fact necessary, it is necessary in every possible world. To deny such principle we would have to come up with an example of proposition that is in fact (logically) necessary but might have been contingent if things had been different. Is it possible to find one? Take for example such necessary truths as

(17) No one is greater than himself

or

(18) All bachelors are unmarried

and imagine which state of affairs would have to obtain in order to make them contingent. One possible argument is that the fact that we use 'greater than' or 'bachelor' is a contingent one and therefore So we might have used those expression in such a way (e.g. 'bachelor' to mean 'inexperienced young men') that would make propositions like (17) and (18) contingent. There is however one flaw in the argument of this kind. Propositions are intuitively aptly understood as contents of statements. Now the shift of meaning of words can of course bring about that the same statement expresses different proposition. And this is exactly our case. Of course if we use 'bachelor' with the new meaning we express a contingent (and probably also false) proposition.

This however in no way implies that the proposition expressed by (18) is itself contingent. As a result transitivity should be accepted as a sound principle for logical modality. If transitivity is granted what shall we say about symmetry and the corresponding axiom $\Diamond p \rightarrow \Box\Diamond p$. The axiom says is that if a proposition is possible in one possible world it is possible in all possible worlds. The counterexample would be to produce a proposition that is for example actually possible but would turn impossible if things had been different. The solution is the same as above. Consider for example the false statement

(19) G.W.Bush is bald.

We do not deny that (19) could express impossible proposition but the proposition it actually expresses cannot be impossible at any world.

As a result it seems reasonable to accept reflexivity, symmetry and transitivity as features of accessibility relation governing logical necessity. But relation that is reflexive, symmetrical and transitive is so-called equivalence relation. Equivalence relations have the distinctive feature that group objects into so-called equivalence classes. And our equivalence does the same – only that it groups all possible worlds into one huge equivalence class and thus makes them all accessible from each other and in a sense equally alternative. And because all worlds are equally alternative we can drop the accessibility relation and stop talking about accessibility and relative possibility. This also matches the intuition that broadly logical modality is the broadest notion of modality at all. Therefore all worlds should count as valid logical alternatives to reality.

4.3. Possible worlds – What They Can Be

In sections 4.2.1 - 4.2.4 we have argued for some principles that are closely related to the problems of modalities. We have however tried to argue that these notions are independent of the problem of modalities. As a result they should be considered to be more fundamental than our conception of modalities and as such should have also some kind of priority. What I want to say is that if we devise a theory of modalities that is in conflict with them, we should modify the theory in question rather than drop any of the basic notions. Therefore we will use them as a guidance in the pursuit of suitable system of modalities. To recapitulate let me to mention briefly the main points that we have established so far:

(i) Actualism: Everything that exists, exists actually. That means that there are no primitive non-actual possible worlds. There are also no primitive non-actual individuals. That does not of course preclude the possibility to "construct" the merely possible out of the actual.

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[118] To remind us reflexivity is represented by $\Box p \rightarrow p$, transitivity by $\Box p \rightarrow \Box\Box p$ and symmetry by $\Diamond p \rightarrow \Box\Diamond p$.

(ii) Haecceitism: Trans-world identity is both a semantically and metaphysically primitive notion. There also can be qualitatively indiscernible possible worlds that differ only in what they say de re about individuals. This however holds only for the actual individuals. For the merely possible individuals we choose the Actualist approach and individuate them as bundles of properties. This also indicates that there cannot be qualitative indiscernible worlds that would differ only in what they attribute de re to the merely possible individuals.

(iii) Absolute possibility: We prefer such notion of broadly logical possibility that is absolute in the logical space rather than relative only to certain possible worlds. Therefore the corresponding theory of modality should use the frame where the accessibility relation is an equivalence and as such can be dropped from the theory at all.

There are also some that we have not explicitly argued for but which we would like to mention as well

(iv) Essentialism. We would like to remain agnostic about whether there are or there are not qualitative (both relational and non-relational) essential properties. Maybe there are, but we are hardly in the position to know. We are inclined to be skeptical and rather claim that there are not any and that much confusion about them originates from conflating properties of individuals and properties of "roles" they play in the actual world. Anyway we would prefer not to commit to the existence of such entities.

(v) Quantification. We assume without argument that the closer we stay to the standard quantification theory the better. It is advisable that we stick to standard objectual quantifiers instead of some modified version of them. Although we might let them to range over weird objects, the quantificational part of them is at least clear from non-modal cases.

(vi) Realism. We would like to take a quite strong realist (or Platonic, if you wish) attitude towards universals. We believe that universals (relations, properties) are one of the building blocks of the reality. Universals are further abundant. Concerning properties there are at least as many properties as all sets of individuals. We also believe that the realm of properties is closed under logical (not, or, and etc) and episthemic (believe that) constructions. As a result some properties might be uninstantiated.[119]

We are persuaded that points (i) - (v) represent preferred principles of forming a theory of possible worlds that as such they should be respected. Point (vi) however represents a controversial doctrine which can be a prima facie reason for rejecting any theory committed to it. We are unfortunately not

[119] One might think about alien properties (properties of alien beings completely unlike anything we know or imagine) but it is not necessarily to go that far. Certainly there are pairs of properties P and Q, where P and Q are instantiated, but the property gained by the operation of conjunction, $\lambda x(Px \& Qx)$ is not. As an example can serve any pair of properties where one applies to concrete (blue) and the other to abstract objects (to be greater than 1). Concerning non-composite properties, all properties of abstract objects (e.g. being perfectly round) are not instantiated in time and space

in a position to defend it here (if it is possible to deliver some conclusive defence all). We can only call attention to the fact that it is a standard assumption in contemporary mathematics and set theory. There we also work with abstract objects, such as sets or numbers and presuppose their actual existence. Of course, a theory that could avoid the commitment to (vi) while satisfying (i) - (v) should be given a preference.

Let us start with a simple question: What do we mean and what do we imagine when we say that something is possible? Take for example me. I could have finished this study a month ago and so avoid the stress before deadline. When I think about this possibility I imagine a situation similar to those I experience every day that includes me, this study and maybe some other objects, further my being relaxed, study being completed etc. And I believe that such situation might have been actual. So what I have in front of my eyes is not a constellation of objects but rather a situation. Also the whole world is composed of situations. These are of course actual. And so the conceivable alternatives to it must be composed of situations (or their surrogates) as well.

We think that the most intuitive account of modalities that respects Actualism and with the view that the actual world is a collection of facts rather than objects o is given in terms of combinations. On such a view (basic or atomic) facts like 'my being a human' are either taken to be composed out of objects (individuals) and universals (relations, properties) or as primitive. In the latter case individuals and universals can be understood as useful abstractions from facts. The idea that possibility is mainly a recombination of some basic ontological building blocks is said to originate from Wittgenstein and has been recently defended by Skyrms (1981) and Armstrong (1986). Because Armstrong's proposal is much more detailed I shall use it for demonstration.

For Armstrong the basic building blocks of the world are simple individuals and simple properties and relations. The simplicity of individuals is constituted by the fact that they have no other individuals as proper parts. The simplicity of properties and relations means that they cannot be analyzed into more basic properties and relations. Properties and relations are further understood as universals, i.e. they can be instantiated by more than one individual. Further universals can be produced by conjunction (but not disjunction or negation or any other logical operation) of simpler ones. There are also no universals that are not instantiated in the actual world. We can of course build complex predicates and relations. So if P and Q stand for universals then predicates like $\lambda x(Px \vee Qx)$, i.e. either P or Q, can be truly ascribed to some individuals but no corresponding universal exist. Armstrong's building blocks are essentially abstractions from states of affairs or facts. It means that "while by an act of selective attention they may be considered apart from the states of affairs in which they figure, they have no existence outside states of affairs"[120] Further the question

[120] Armstrong (1986), p. 198 (Page reference of this article is the reference to the reprint in Tooley (1999b).)

on the nature, number and features of these building blocks is essentially open. It is not up to logic, but up to science to determine which atoms there are (it there are any atoms at all). We understand this so that from the viewpoint of logic atoms in every category have to be selected similarly to Boolean sets of logical connectives. The set of simples of some sort has to be something like a set of "generators" from which one can "construct" the rest of the entities of that sort. As a result we can see that Armstrong's ontology is quite modest. The only posited entities are states of affairs.

What are now possible worlds? Armstrong begins with the notion of atomic state of affair, which is a state of affair that contains only simple individuals and universals. Similarly to universals states of affairs can be combined into new (complex) states of affairs by conjunction (but not by other operations). Such a conjunction is called molecular state of affairs. The world is then (possible infinite) conjunction of states of affairs. All the states of affairs are actual. The notion of the possible world is then introduced semantically. An atomic statement corresponds to the atomic fact and formally is of the form '$P(t_1,\ldots,t_n)$', where P stands for n-ary predicate and $t_1,\ldots,t_n$ are singular terms. Take for example statements 'Fa' and 'Ga' and suppose that there is some a that is F but not G. 'Fa' is then a true atomic statement and describes some state of affairs, namely 'a's being F'. On the other hand 'Ga' is false and fails to describe anything. "But we can also say that a's being G is a possible (merely possible) atomic state of affairs. A merely possible state of affairs does not exist, subsist, or have any sort of being. It is no addition to our ontology. But we can refer to it, or, better, make ostensible reference to it."[121] Possible world is now produces as two-step combinations. First of all, every combination of simple properties, relations and individuals yield possible molecular state of affairs. These states of affairs are then combined in all ways and produce possible molecular states of affairs. Possible world is finally a possible molecular state of affairs that "is thought of as the totality of being".[122] Such possible world represents aptly our intuition that it is possible that the actual individuals could have different properties or stand in different relations. But what about existence of individuals? It is intuitive to suppose that I might not have existed or that there might have more individuals. Armstrong solves it by allowing for contracted or augmented possible worlds. A contracted world is such where some individuals or universals are missing, augmented is one where there are extra elements (measured probably by the actual world). Concerning individuals he goes for what he calls "weak anti-Haecceitism", a doctrine that individuals do not have thisnesses but "qua individuals, are merely, barely, numerically different from each other"[123]

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[121] Ibid., p 199.
[122] Ibid.
[123] Ibid, p. 204. Armstrong claims further that thisnesses would make the addition of individuals impossible. Each new individual would have to posess a thisness. But thisness (as usually understood) is ontologically dependent on the particular individual. (Note that for Armstrong haecceities as we have talked about them in section 4.2.3 are not acceptable.) So thisnesses of these new individuals that do not exist cannot be present in the actual world. To posit them would mean to violate the principles of Actualism.

As such, additional individuals can be added "by analogy" to the actual ones. Concerning universals Armstrong has a stronger view: there cannot be but universals instantiated in the actual worlds. So called alien properties or relations are not conceivable (in the strongest possible sense) and no augmentation is possible. The limist of what is possible are thus set by the actually instantiated universals. Contracted worlds are on the other hand no problem. We simply take the simples for contracted worlds as subsets of the actual ones. Possible augmented or contracted worlds are then (presumably) created again by recombination.

We find Armstrong's theory of possible worlds (or at least its core) very attractive. It respects the intuition that possibility rests on the notion of combination, respects Actualism and has a quite modest ontology. Its quantifiers are ranging over everything there is so it does not posit any primitive non-actual entities nor does it leave some entities out of the range of quantifiers. Although it denies the existence of individual thisnesses, it has concerning possible worlds similar results as Haecceitism. Identity and trans-world identity is taken as primitive and from the viewpoint of Haecceitism it does not conflate any possibilities concerning the actual individuals. Another advantage is that if the combinatorial part of possible world construction can be carried out "mechanically", Armstrong can make do without primitive notion of possibility. That is why Armstrong pays so much attention to the choice of simple universals and allows only conjunction in construction of complex ones. He thus tries to avoid the situation in which he could get contradictory states of affairs into one possible world. But whether the simples can really be chosen so that they are perfectly independent is another question. One more advantage could be that its formal models can be Kripkean i.e. with varying individual domains and as such do not necessarily validate Barcan formula, its converse, or thesis of necessary existence of individuals.

Armstrong's Combinatorial theory of modality (ACT) faces nevertheless serious difficulties which according to my point of view originate mostly from its modest ontology. Let me briefly mention some of them. First of all it violates the principle of absolute possibility (our (iii) above). If we remember the accessibility relation we talked about above, we stressed that it should be an equivalence relation in order to get a notion of necessity and possibility that is not relativized to the actual world and thus does not allow for contingently necessary propositions. Imagine now two similar possible worlds w_1 and w_2 such that 'Fs' is part of w_1 but F (a simple property) is not among building blocks of w_2.[124] World w_2 is accessible from w_1, now problem there. But from the standpoint of w_2 F is an alien property. And because it is not instantiated at w_2 it is (from the viewpoint of w_2) necessary nonexistent. And so w_1 cannot be admitted as possibly relative to w_2 and the relation of accessibility has to be asymmetric. As a result necessity and possibility cannot be

[124] Argument originates from Lycan (1999), p. 35, who refers to Philip Quinn.

considered as absolute but merely relative to some possible world. This objection is also closely related to Armstrong's view about universals. His ban on uninstantiated universals seems to impose serious limit to what is possible. This is the case of alien properties – properties that are not instantiated in the actual world nor can be constructed out of the actual. Similar remarks concern non-atomistic worlds, worlds that operate according to completely different physics etc. Armstrong can of course bite the bullet and insist that what is impossible according to ACT is really impossible. But this "impossible" must be taken in the strongest possible sense – in the sense of logical impossibility, i.e. in the same sense in which 5 is both prime and not prime. But there is nothing logically inconsistent on there being alien properties. One can insist that it is impossible in very strong sense but there is nothing contradictory ion it. Moreover there are many people who at least believe in alien properties, irreducible egos and the like.

Another really serious problem for ACT is that its possible worlds exist as a mere fictions.[125] Possible states of affairs have no kind of being at all. But how can we then take the semantics to express anything about real nature of modality? We have urged in section 4.1 that we must take the informal semantics of modal notions seriously. But Armstrong's fictionalism is certainly not what me meant by that. Moreover, if possible worlds are fictions that it is difficult to explain the quantificational part of ACT. What are the entities that belong for example into the ranges of quantifiers? Certainly not individuals as we know them form the actual world. There is also the mysterious issue of bare individuals (and above all bare merely possible individuals) and their identities and non-identities, which especially in the case of merely possible individuals violates our Haecceitist principles that non-actual individuals cannot have thisnesses and thus be numerically distinct from other indiscernible non-actual individuals.

For this and many other reasons[126] ACT is too weak to cover our intuitions about modalities and should be abandoned. It has however a sound core and we believe that many of its deficiencies can be "cured". To avoid Armstrong's fictionalism we can follow the proposal of Kim (1986) and treat combinatorial possible worlds as actually existing abstract entities (possible states of affairs can be represented as for example ordered pairs and possible worlds as set-theoretical constructions out of them) resembling in their mode existence the mathematical objects. This would bring us closer to "serious" semantics of modal notions. We can also release the ban on uninstantiated universals and take rather Platonic view on properties and relations. According to it there is an universal for every predicate and relation and the realm of properties is closed under logical (not, and, or) as well as epistemical (believes that, knows that) operations. This would save us from the problems with symmetry of the accessibility relation and relative possibility as well as the problems

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[125] Armstrong (1986), p. 210.
[126] For other objections see for example Lycan (1999) section XI or Kim (1986).

with alien properties. We would be forced to give up Armstrong's neat ontology but we would still stay within limits set by the (sound) principles located at the beginning of this section.

One system that according to us draws its motivation from ACT is the Transparent Intensional Logic (TIL) of Czech logician and philosopher P. Tichý. TIL is a sophisticated intensional system that is not concerned primarily with modal logic but it uses very interesting apparatus of possible worlds which is worth considering. Tichý[127] makes also use of the combinatorial conception of possible worlds. He accepts the idea that world is a totality of facts rather than things. To define possible worlds Tichý uses several auxiliary notions. First is the one of determiner. A determiner is an abstract intensional entity that singles out some object of given category. It is aptly characterized as function of possible worlds (and times) to certain category of objects. We describe determiners by expressions of our language, but determiners are not identical with them. So we can say that *G. W. Bush* or *the President of the U.S.* are determiners. In that case we do not mean the expressions themselves but the determiners behind. (To avoid conclusion we will write determiners in italics.) Every fact can be, according to Tichý, correlated to some determiner. "To each fact there corresponds a determiner in such a way that the fact consists either in the determiner's singling out a definite object or in its failing to single out anything at all."[128] So the fact that G. W. Bush is the President of U.S. consists in singling out of G. W. Bush by the determiner the *President of U.S.*. It is however contingent fact that the determiner does so. It might have singled out Gore or anyone else as well. Besides individual determiners there are also determiners for propositions, properties. Simply said behind (almost) every expression of the language there is a determiner of some sort. Another notion we need is the determination system. A determination system is an assignment of objects (of appropriate kind) to some determiners. It is one-to-many correspondence that links (some) individual determiners with individuals, some truth-value determiners with truth-values, etc.. Not every determination system is, however, a possible world. Determiners are not mutually independent and so for example no determination system where some individual is both entirely red and entirely green is realizable. But possible worlds are only realizable determination system. But if possible worlds are determination systems, so it the actual world. The actual world is in TIL nothing more than an abstract (mathematical) object.

Another important feature of TIL is that quantifiers range over constant domain of "bare" individuals. The reason to introduce constant domain is motivated by the view of Kant, Frege or Russell that existence is not a predicate of individuals.[129] We will leave this issue aside and focus on

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[127] Tichý (1988), p. 177 - 186

[128] Ibid., p. 178.

[129] For Tichý's presentation and argument see Ibid., p. 180. The main argument consists in he fact that there cannot be any non-trivial existence test or criterion for individuals. Theories of possible worlds with varying domain pressupose

bare individuals and their identities and non-identities. Bare individual is an individual that has only trivial properties.[130] It does not mean that it would actually lack all non-trivial properties. In fact, in the actual world, it always has some non-trivial property and so we can never meet it completely "naked". We only says that no non-trivial property is being constitutive for its identity. Similarly to Armstrong, bare individuals qua bare individuals are simply distinct form each other. Tichý further claims that individuals have no essential properties. Because individuals are primarily bare they can have and lack any non-trivial property. So the very same individual can be person in one world, pig in another, hammer in third and I do not know what in fourth. (Well, who knows. The adherents of Hinduism would be certainly pleased to hear that.) But the same holds in time. One individual can be in the same world person at time t_1, pig at t_2 and hammer at t_3. Of course, the actual world is not the one where such mutations occur. But it is only a contingent fact. There is nothing in the nature of individuals that would prevent such changes. But how does Tichý explain what we mean when we say that something does not exist? The key notion is again the one of determiner. Individual determiners can be divided into two classes: those that succeed in determining an individual at some w and those that do not. The examples in the actual world are *the President of U.S.* for the prior *and the winged horse* for the latter group. So what is being classified by existence and non-existence are determiners and not individuals.

In comparison to ACT is TIL concerning the notion of possible world theoretically much more suitable. They both share similar (but not uncontroversial) understanding of individuation of individuals, respect Actualism and provide a combinatorial account of modality. TIL however does not have any problems with augmented or contracted worlds and is therefore governed by preferred S5 logic. As a result it also has as theorems the unwelcome Barcan formula (BF) and its converse (CBF)[131] and the thesis of necessary existence

(NE) $(\forall x)\Box(\exists y)(x=y)$.

But they are compensated by extreme anti-essentialism or as Lewis calls it extreme Haecceitism, i.e. thesis that individuals have no non-trivial essential properties. Using this principle together with constant individual domain renders BF harmless. For if it is possible that there are aliens, then there naturally is some actual individual (all individual are recruited from the actual world) that is an alien in some possible world. There is no essential property to preclude this. Also turns out to be trivial. Tichý, however, coins some other notion of existence that applies to determiners and thus has an explanation for our existential statements in the natural language. So we are still able to

that it meaningful to say that an individual does or does not exist in a given world. As such they are incompatible with this view on existence and thus with TIL.

[130] Ibid., p. 210. Tichý says there that a bare individual can lack any of its non-trivial properties and remain the same individual. Non-trivial properties are not constitutive to it. So bare individual is an ordinary individual devoid of all non-trivial properties. In other words bare individual has only trivial properties.

express the idea that Pegasus does not actually exist. It only means that in the actual world the determiner *Pegasus* singles out no individual. Another advantage against Armstrong is that possible worlds are full-fledged abstract and actually existent objects and not only fictions. Yet another is that given the fixed and all-inclusive domain of individuals and properties we get no problems with alien properties and alien individuals. There is however one point where ACT wins. We have seen that not every determination system is a possible worlds. Possible worlds must be realizable. But the notion of realizability is a modal one and so TIL is definitely stuck with primitive possibility. ACT on the other hand at least promises that if the combinations can be done mechanically we could make do without primitive modal notions. If it can be proved that this cannot be done that TIL and ACT are in this respect on a par.

The general trade-off for the theoretical advantages is the conception of bare individuals and their individualization, Fregean conception of proper names, extreme anti-essentialism and necessary existence, each of these being a very controversial issue. Let me explain. Concerning bare individuals there are many problems. I shall not mention them there but one classical objection is that the notion of a thing that has only trivial properties is logically inconsistent. The strategy is to ask whether the property 'to be an object that has only trivial properties' is itself trivial or not. In both cases we get paradoxical results.[132] Also the anti-essentialist intuition is difficult to believe. Many philosophers have for example argued for necessities of origins or essential properties of various kind. Possible defense is to reply that if we are concerned primarily by logical possibility we have to be ready to envisage weird worlds. We admit. But wow weird van these worlds be? Aren't there some limits after all? Take for example G. W. Bush. In which sense is it possible that he is a musical performance? If not, why can the "bare" Bush be for example hammer but not musical performance? And there are many other disturbing questions. We also proclaimed above that it is preferable to remain neutral on this very controversial issue. But TIL is not neutral and relies heavily on the anti-essential intuitions. If anti-essentialism is false, there will not be enough actual individuals to produce all the possibilities (e.g. there being aliens). Certainly neutrality on the essentialism - anti-essentialsim issue would be better. At least, it would not force us to proclaim that individuals must go in one world-history through weird uninterrupted sequence of having various properties, e.g. being a person and immediately afterwards become a hammer.

We further mentioned the treatment of names. Above we have argued for more or less direct reference view and said that it is one important component of primitive identity of individuals and above all primitive trans-world identity. According to Tichý names "express" individual determiners. So the expression 'G. W. Bush' expresses a determiner which picks out some

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[131] For precise formulation of these theses see section 3.2 above.
[132] For complete version of this argument see Kolář (1999) p. 118 – 121.

individuals at some worlds. Determiner is in this respect similar to Fregean sense. This means that the reference of a name is not determined directly but by some associated sense. Senses of names are also problematic entities. They are usually understood as being describable by definite descriptions. This leads to the descriptive theory of names. But Kripke (1996) has persuasively shown that such theory is inadequate. Let us state just a short motivation. When we baptize an infant Tichý wants us to believe that even though we give the name directly to the child (while touching his forehead) we in fact attach the name to some abstract determiner that contingently singles out this child. For us it is difficult that this gives a real picture of how name function.

Lastly, NE certainly violates our intuitions that things, animals and people come into being and pass away. But according to TIL my ceasing to be only means that I loose some properties (be a human being, be a person) and gain some other (e.g. be a typewriter) The only difference between the world where I am a living human and where I am a typewriter is that in the former the determiner *Ondřej Tomala* singles out me, but in the latter does not pick out anything. This view does not have to be a priori wrong. We only think that there are too many counterintuitive consequences. One is that in the world where I am a typewriter the statement

(20) Ondřej Tomala does not exist

as we normally mean it would be true but I, the very same individual, would be present among the individuals of that world although disguised as a typewriter. My name would abandon me. But my anti-essentialist intuition is that no matter what change I undergo, I will still be Ondřej Tomala. You would naturally say that our possible world is a world where the individual called Ondřej Tomala in the actual world is a typewriter. But in TIL 'actual' is an indexical and so it cannot be used to differentiate between worlds. On the other hand, if we let *Ondřej Tomala* rigidly refer to the some individual o then in some possible world we would get weird statements as true.

(21) Ondřej Tomala exists and is a typewriter

is one of them. A defender of TIL would object that the logical 'exists' is trivial and that in (21) we used different sense of 'exist'. It is certainly the latter sense that is the one used in ordinary talk. But then we need logical analysis of this non-trivial sense of 'exist' and the whole problem of logical analysis of existence starts again.

The conclusion is that TIL certainly presents a very sophisticated theory of possible worlds. It is devoid of many theoretical problems but we have to pay by wild and maybe even too wild ontology. It entails strong anti-essentialism, we are stuck with controversial bare individuals, and its theory of names is descriptivist. Any of those features is a more than good reason to reject it. So neither ACT nor TIL are able to satisfy our intuitions about modality. Does it mean that we have to abandon the combinatorial approach and think of possible worlds as things rather than complex states of affairs. We are not in position to answer it now. One possible answer is to reject TIL and accept some more realistic version of ACT and concede that it is not full-fledged theory of possible worlds but only model or representation thereof. But we want a full-fledged theory of possible worlds. So we are forced to abandon the combinatorial approach and consider some competing theory.

Most of competing Actualist theories construct possible worlds as sets of states of affairs or propositions (or conjunction of its members).[133] The most common problem of such doctrines is that they either cannot adopt enough fine-grained view of states of affairs or propositions and at the same time claim that there is unique actual world. The dilemma is following. If we have theory with fine-grained states of affairs we can distinguish necessarily equivalent states of affairs. In such theory p and p & (q ∨ ¬q) are not identical states of affairs. We can make for example appeal to their surface structure and say that p is simple while p & (q ∨ ¬q) is not. Now take some states of affairs p_0 that is according to the theory in question also a possible world. Then p & (q ∨ ¬q) is also a possible world and we have distinct copies of the same world, which is counterintuitive. This is in particular problem in the case of the actual world because we get not one but many of them. Actualist systems are also forced to admit that possible worlds can be described by propositions that do not exist in them.[134] This is especially the case of singular propositions (propositions of the form 'Pa' where a is individual and P is a predicate) about individuals that do not exist in some world. So in the world where G. W. Bush is missing there cannot be the proposition 'G. W. Bush does not exist' but this proposition nevertheless true about the world somehow.

In what follows we would like to discuss an alternative to these views, the system of abstract possible worlds presented by Zalta alone (1993) and jointly with Linsky (Lisky and Zalta, 1994). Linsky and Zalta come with an original proposal of Actualist system, which can be formulated in so called simplest quantified modal logic (SQML). SQML is basically system S5 with (i) distinguished possible world, (ii) constant individual domain, (iii) identity, (iv) standard quantificational theory and (v) without non-denoting singular terms. SQML is normally understood as a possibilist system and as such Actualists reject it. Linsky and Zalta, however, provide it with an original Actualist interpretation.

The crucial notion for the Actualist reading of SQML is the notion of contingently non-concrete object (CNC). The distinction between concrete and abstract is normally understood as absolute. If an object is abstract, it is essentially abstract and if it is concrete, it is essentially concrete. Linsky and Zalta find such a view unjustified and propose to add third category of objects – objects that are actually abstract but might have been concrete. Their existence and properties are the same as those of numbers, sets and other abstract objects. They differ from essentially abstract

[133] For example Plantinga (1982), Adams (1974), Kaplan (1975).

objects only with regards modal properties. Contingently non-concrete objects are of course a controversial part of one's ontology. However Linsky and Zalta think that most Actualists should find contingently non-concrete obects acceptable, "since in various ways, they already invoke objects that straddle the alleged categorial divide between the abstract and concrete."[135] Atualist really posit such entities, i.e. thisnesses, singular propositions or abstract states of affairs containing concrete individuals. As a result, contingently non-concrete objects should be acceptable at least as much as these entities.

The proposal to breach the barrier between concrete and abstract objects is certainly a very radical step. So let us stop here and describe the proposal in greater detail. We will do so through theory of abstract objects of Zalta (1993). According to Zalta there are two kinds of properties – intrinsic and extrinsic - and as a result also two modes of predication – standard exemplification (captured classically by the formula 'Fx') for extrinsic and so-called encoding (captured by new notation 'xF') for intrinsic properties and relations. Ordinary objects like atoms, cats, people have only extrinsic properties. Zalta's theory however announces that there is special subclass of objects that have both intrinsic and extrinsic properties and thus both exemplify and encode properties. Encoding and exemplification are independent and so these special objects can encode different properties than those that they exemplify. Encoding is however (unlike exemplification) limited to only unary properties. Exemplification is basically the notion that we know from standard logic and is governed by classical principles like the Law of Non-contradicton or Law of Excluded Middle. But how shall we understand encoding. First of all, only abstract objects can encode properties. Properties encoded by an object are those that are constitutive to its nature and its identity. Moreover, an object can exemplify and even necessarily exemplify large number of properties and relations. But these have no effect on its nature. Zalta gives following motivating examples for encoding.[136] First, consider number 1 (treated as an object) of number theory and consider its various properties. This number contingently exemplifies properties like being thought about by me right now or being denoted by the numeral '1'. It further necessarily exemplifies properties like having no location in space, having no shape. But these properties have in fact nothing to do with the nature of number 1. On the other hand, properties like being odd, being prime, being the successor of 0 have serious impact on its nature and its identity. They constitute what number 1 really is. So we need a concept of predication stronger than necessary exemplification. And so we need encoding. So the just mentioned properties are then said to be encoded by number 1. Consider another object, the famous Sherlock Holmes. According to Zalta, Sherlock Holmes encodes all the

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[134] See for example Adams (1981), p 20 – 22.
[135] Linsky (1994), p. 432.
[136] Zalta (1993), p. 398.

properties ascribed to him in the novels of Conan Doyle. These include being a person, being a detective, living in London, those properties that can be reasonably derived from them and many others. These properties constitute, so to say, the nature of Sherlock Holmes. Holmes also exemplifies many other properties. Some of them contingent, e.g. being fictional, being an inspiration to modern criminologist, some necessarily e.g. not being a person, not living in London, not being a spoon (in the same sense ordinary objects exemplify them). The encode - exemplify distinction is also extremely useful in case of impossible objects. Impossible objects are according to Zalta abstract objects that are necessarily abstract. Their identity is thus constituted by properties they encode. So for example the round square encodes squareness and roundness and nothing more. It, however, does not exemplify any of those. So there is a sense in which we can say that the round square is (i) incomplete and (ii) is round and square. But we can keep the ordinary predication for ordinary objects. Concerning the exemplification, every object has to be perfectly determined in respect to every property or relation and no inconsistency is allowed. So on the other hand we can still keep the intuition that in the ordinary sense it is not round nor square because it is necessary non-concrete object.

Now we are in position to define the individual domain of system of Linsky and Zalta (LZ). It contains two sorts of objects: ordinary objects O and abstract objects A. All of these exist and are actual. There are no shadowy objects of any kind. Members of O are ordinary concrete objects like people, animals or things. Note that concrete is for Zalta synonymous with spatiotemporally located. Abstract is then the negation of concrete, i.e. non-concrete. Although Zalta's principle of individuation is different we spoil nothing if we assume that concrete objects have thisnesses. Abstract, i.e. non-concrete, objects are of two sorts. There are contingently non-concrete and necessarily non-concrete objects. Into the latter group fall for example number 1 and the round square. Into the prior Pegasus and my future son. Necessarily non-concrete objects are the familiar abstract particulars. Contingently non-concrete objects are objects that are not concrete in the actual world but are concrete in some other possible world. Abstracts objects are further individuated by encoded properties. As a result there cannot be indiscernible abstract objects. Exemplified properties then play no role in their numerical identity.

Last thing we need is the definition of the notion of possible world. LZ is an Actualist system so non-actual possible worlds cannot be taken as primitive. Therefore we have to construct them out of what is actual. Zalta's ambition is to provide not a model but a metaphysical theory of possible worlds and thus he does not want to construct them as mathematical or set-theoretical objects. The central notion of LZ is the state of affairs (SOA). States of affairs like *Bill's loving Mary* either obtain or they do not. Zalta's trick is to introduce so-called SOA-properties. These are properties that are closely linked to states of affairs and which are exemplified by objects when these states of affairs obtain. If we take two states of affairs *Bill's loving Mary* and *Mary's not loving Bill* (represented by formulas L(b,m) and ¬L(m,b)) we get the corresponding properties being such that Bill loves Mary and being such that Mary does not love Bill by abstraction as [λy L(b,m)] and [λy ¬L(m,b)]. Although these predicates seem weird they are perfectly all right nd well behaved. Necessarily an object x exemplifies property being such that Bill loves Mary iff Bill loves Mary. As a result we get perfect correspondence between states of affairs and SOA-properties. The problem is now that if certain state of affairs obtains, then every object exemplifies the corresponding SOA-property. (Variable y in λ-terms is vacuously bound and so we cannot apply the predicate exclusively to some objects but have to apply it to all or none.) Therefore we have to use the logic of encoding, which is governed by different principles. SOA-properties can be encoded by abstract objects and thus serve to distinguish some special sorts of them. Note that metaphysically it is a mater of fact whether an abstract object encodes some SOA-property. Another notion is the one of situation. Situation is an abstract object that encodes only SOA-properties. In other words, it is intrinsically characterized by states of affairs. Formally we can say that

(22) $Situation(x) =_{Df} A!x \;\&\; (\forall F)(xF \rightarrow (\exists p)(F = [\lambda y\, p]))$

where A! stands for being abstract, F ranges over properties and p over states of affairs (or propositions if you wish). Further for every group of SOA-properties there is a situation which encodes all and only those properties. This assures that there is for example object that "is" such that Bill loves Mary. It is simply the one that encodes the corresponding SOA-property. Because situations are abstract they are individuated by encoded properties and thus there are no indiscernible situations. Concerning exemplification, it is left open which properties does any situation exemplify. Our last situation can for example exemplify property of being disturbing to Mary's husband.

Another important notion of the theory is now that certain state of affairs is factual in situation s. It is apparent that state of affairs is factual in s iff s encodes the corresponding SOA-property p. In Zalta's notation $s \models p$.Worlds are now just special kind of situations. Worlds similarly to situations have internal and external properties. The internal properties are those SOA-properties that characterize what is going on in the world and which identify the world among others. The difference is that worlds are maximal. So situation s is a world iff it is possible that all and only factual states of affairs are factual in s. Formally

(23) $World(s) =_{Df} \Diamond(\forall p)(s \models p \equiv p)$.

There are many other things that can be said about worlds and situations but it would takes us too far from our topic which is the discussion of modalities. Let us, however, make few concluding remarks. Zalta's theory of objects provides us with good basis for possible worlds semantics. It designs a notion of abstract possible world based on states of affairs and situations. It also provides suitable abstract surrogates for non-actual objects. Slight but not serious deficiency at this point is that it takes one modal operator as primitive. It is time to use LZ "in practice" and see how it can be help with the actualist semantics for SQML.

We start with set of possible worlds with a distinguished actual world and a domain of objects that is, as we know, partitioned and contains actually concrete, contingently non-concrete and necessarily abstract objects. The system is governed by S5 logic so we do not need to mention any particular accessibility relation. Because Linsky and Zalta choose S5 they have to defend BF, CBF and even NE. With the notion of contingently non-concrete objects we get quite intuitive defense of BF. What BF requires is that if it is possible that x is F, then there is an x which is possibly F. We have to realize that BF does not require that there should be something that is actually F. It only requires that there is an object, no matter which, that is F in some possible world. But aren't our contingently non-concrete object just ideal entities for this purpose? Indeed, they are exactly what BF requires. Although they do not exemplify some F in the actual world they do so in some possible one. So BF is given interpretation that is perfectly compatible with actualism.

What about augmented and contracted worlds and necessary existence? First of all note that there are two notions of existence in LZ. One is expressed by the existential quantifier. That is the strong existence in the Actualist sense. In this sense everything exists and because LZ accepts constant individual domain for all possible worlds, everything that exists exists also by necessity. The other notion is the intuitive kind of existence. To exist in this sense means to be concrete, i.e spatiotemporally located. In LZ this sort of existence is represented by simple predicate 'E!'. This second notion is supposed to capture what we normally mean by 'exist'. When we say that Pegasus does not exist we mean that there is no concrete object that exemplifies all properties we attribute to Pegasus. We certainly do not search for it among abstract objects. So it is the second notion that we normally use for existence and non-existence claims and thus 'E!' rather than sole '∃' should appear in the logical analysis of such statements. Frankly said, what is the difference between the world where for example G. W. Bush does not exist at all and the one where he exists but fails to be concrete. Such understanding of existence now makes the account for augmented and contracted worlds quite straightforward. Both concrete and special objects exemplify properties in the same way (abstract objects moreover encode them). So there is no difference between them with regards to the qualitative part. Therefore the only difference concerns the spatiotemporal location, i.e. being concrete. That's why the set of objects existing in w is simply determined by the extension of E! in that world. And because we have the category of contingently non-concrete objects the extension of E! can vary from world to world.

Notice also that LZ is completely Actualist. Non-actual worlds are defined in terms of actually existing (but non-obtaining) states of affairs and corresponding SOA-properties. Non-actual "possibilia" are actually existing abstract objects. All quantifiers range over all actually existing objects and they range directly over these objects. In other words, they are objectual. Even the meta-language of LZ does not make appeal to any non-actual entities. BF, CBF and NE are also given an Actualist interpretation. What about the other preferred features listed at the outset of this section? We can see that LZ satisfies criteria (i), (iii) and (v). LZ is also within limits set by (vi). What about the rest? Concerning (iv), i.e. essentialism, we have to reconsider the definition of essential property and separate the parts concerning possibly concrete, i.e. ordinary concrete and contingently non-concrete, objects and necessarily abstract objects. If a is necessarily abstract object then we say that P is essential property of a if necessarily a exemplifies P. If we, however, need the notion of essential property of ordinary concrete and contingently non-concrete objects than we can keep the usual notion: P is essential to x iff necessarily, if x is concrete, then x is F. So the system is ready to accommodate essentialism or anti-essentialism concerning ordinary concrete and contingently non-concrete objects if required but it is not committed to either. And what about (ii)? As we have seen ordinary concrete objects are qua individuals numerically distinct and thus there are also all de re possibilities about them. Contingently non-concrete objects are nevertheless individuated qualitatively, although not with respect to exemplification but encoding. But it is encoding that is important to the nature and identity of these (abstract) objects. It is, however, impossible that there be two abstract objects with the same intrinsic characterization. So there should be no distinct possibilities for indiscernible abstract objects in this sense. Because of the way of individuation of objects we get without any greate problems the notion of primitive transworld identity for ordinary concrete objects. The rest of objects are then identified by their encoded properties. But these objects are immutable with regards to encoded properties, so the identification is also trivial. Lastly, names are rigid designators on the constant individual domain and so LZ is compatible with Kripkean theory of names.

So we have finally reached a theory of possible worlds that satisfies into great extent the beforehand established principles. As far as we can see it surpasses any other Actualist theory in both theoretical and metaphysical isses. Moreover, the underlying theory of abstract objects can be axiomatized and thus given precise theoretical shape (Zalta 1993, 1996 and 1999). The cost is one primitive modality and abandoning of the strict difference between what is abstract and what is concrete. We think it a one that can be accepted.

Concerning the formalization of LZ we already said that SQML is fully appropriate. LZ is in fact a powerful defense of SQML. We will not design SQML here, but we can direct the reader to the system $S5_D$ in section 3.4 only with the qualification that the model is now an ordered quadruple

<W, w_α, D, V> where W, D, V are as in S5D and w_α is the distinguished actual world. It can be also easily shown that such system of modal logic is immune to Quine's criticism that we discussed in Chapter 2. $S5_D$ has a restricted rule of necessitation and adopts free logic concerning non-denoting singular terms. As a result the argument with 9 and the number of planet is not valid in it. We also did not commit to any deep kind of essentialism. We introduced the notion of thisnesses but thisnesses are non-qualitative and incommunicable properties. It is impossible that more than one individual can possess given thisness and every individual possess its thisness whenever it exists. So we are committed to the claim that there are necessary properties, i.e. such properties that if any individual has them it has them necessarily or it is impossible for it to have them at all. This however does not commit us to the view that there are properties such that some individuals have them necessarily and other contingently. But Quine's essentialism is characterized by the latter and not the prior.

Before we conclude this section let us return once more to our combinatorial accounts of modalilies – ACT and TIL. It seems to us that the idea of contingently non-concrete objects could be of some aid to the combinatorial accounts. Take for example TIL. In TIL, we were left with three major problems: anti-essentialism, existence as predicate of determiners and a Fregean theory of names. We believe that the latter is a distinguished feature of TIL that cannot be altered without ruining the elegance of the whole system and the idea that possible worlds are genuine and still abstract. Higher-order existence and anti-essentialism are, nevertheless, expendable. We believe that TIL would become more acceptable if we replaced anti-essentialism by the theory of contingently non-concrete objects. Remember that we got our contingently non-concrete objects by dropping the view that the properties being abstract and being concrete are essential to objects. The introduction of contingently non-concrete objects would thus only be a consequent application of TIL's own anti-essentialist principle. If we allow for contingently non-concrete objects and interpret the informative 'exist' as being concrete, we will not be forced anymore to explain why and how must individuals go through uninterrupted instantiation of sometimes bizarre sequences of properties. They can simply "disappear" from the concrete and thus observable part of the possible world.[137] TIL would get rid of its commitment to anti-essentialism and thus would certainly become more acceptable. Also if the informative sort existence is interpreted à la Linsky and Zalta as being concrete we could restore the view that existence it a feature of individuals. Individuals that do not exist in some world in this sense do not have to be missing. They only lack concreteness. We think

[137] Similar proposal has been recently made by V. Svoboda in (2001). He proposes, however, to treat nonexistent individuals as concrete objects that are nevertheless empirically unobservable. We find abstract objects more appealing than concrete objects that are not spatiotemporal. Svoboda has also some problems saying what does it mean for individual to be "empirically unobservable". While he has to use some extra definitions, we can directly use our notion of contingently non-concrete object.

this modification would bring TIL another step closer to the intuitive view that existence has to do something with individuals, namely, that it makes them concrete.

Last remark concerns the combinatorial systems in general. It seems to us that the we could use the notions of contingently non-concrete object and individuation of abstract objects by encoded properties to solve some problems that combinatorial accounts have to face. For example contingently non-concrete objects would be a very good solution to the problems with augmented worlds which we encountered in ACT. All combinatorialists take properties as basic metaphysical atoms. Now, maybe if we go for partitioned individual domain composed of ordinary concrete, contingently non-concrete as well as necessarily abstract objects we might be able to produce possible worlds, at least into certain extent, as combinations of these atoms. We are however not in position to propose any particular solution at this place. We nwvertheless think it si project worth pursuing.

5. Conclusion

Our pursuit of the reasonable theory of modality is now essentially over. We have seen that philosophical and logical analysis of modalities is a very interesting but also very difficult and demanding issue. Despite its rise in the 20^{th} century very little problems (if any) have been settled and we are still not sure about truth-conditions and meanings of various modal statements. The situation is, moreover, complicated by the vast number of systems of modal logic and theories of possible worlds from which almost ever one suffer from some difficulties. The project is however not impossible. And our present study is a proof thereof. What one need is only to change the methodology and try to look for more support in the non-modal part of reality.

The reason is that historical development of modal logic suffers from the fact was that the speed and productivity of the development of modal calculi exceeded incomparably the speed of development of formal semantics and it again surpassed in its speed the development of intuitive depraved semantics. So in the beginning there was no given set of truths and intuitions that philosophers and logicians were trying to formalize. It was the other way round. Some logicians produced calculi, another were trying to provide it with some model-theoretic semantics and finally philosophers were attempting to make at least some sense of these formal constructions and link them to the intuitive notions of modalities. Interestingly enough these activities were carried out independently on each other. Logicians were not interested in the depraved semantics, philosophers found little appreciation for formalisms.

In our study we wanted to show that a radically different approach has to be taken. It is first necessary to gather some philosophical or intuitive views about modal and especially non-modal features of reality, argue for them independently and first then look for some theory of possible worlds. It is also important to keep the logical, semantical and philosophical issues tightly together. Only then can we find answers to perplexing questions about modalities and their analysis.

Using these principles we first purified the notion of broad logical modality and distinguished it from other philosophical notions as well as from other kinds of modalities. We then rejected possibilism and fictionalism and argued against D. Lewis' extreme realism. Because of the complexity of the issue we split the pursuit of the adequate theory of possible worlds into several steps and tried to argue independently for some views that could serve as component or criteria of such theory. Thus we have argued for Actualism, Haecceitism with respect to the actual individuals for and the notion of absolute necessity and possibility. There we also got first hint that system governed by S5 logic is the most accurate one.

The result of our inquiries is what we consider to be a sound set of criteria that should be fulfilled by any theory of possible worlds in order for it to be sound and acceptable. Our candidate for the position is the system of abstract possible worlds of E. Zalta and his and B. Linsky's Actualist interpretation of SQML. It is in fact one of the few system that covers or can cover all of the principles from outset of the section 4.3. We, however, also proposed improvements to existing combinatorial accounts and envisaged a possible construction of combinatorial system that could replace Armstrong's version.

We believe that by employing the proposed theory possible worlds together with SQML we can get strong and technically elegant theory of modality and a powerful tool for its analysis. We would still not be able to determine truth values of many modal statements but we would at least know what has to be the case for them to be true or false.

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